EXCLUSIVE RARE B DECAYS (FCNC)

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Durham - Lattice Meets Phenomenology - 2010

RICH PHENOMENOLOGY ...

... to test short-distance flavour physics

$b \to q + \gamma$

$B \to V\gamma$ ($V = \rho,\ K^*$)

– Γ

– time-dep. CP asy. $S_{K*\gamma}$

$b \to q + \bar{\nu}\nu$

$B \to P + \bar{\nu}\nu$
$B \to V + \bar{\nu}\nu$

– $d\Gamma/dq^2$

$b \to q + \bar{\ell}\ell$

$B \to \bar{\ell}\ell$

– Γ

$B \to P + \bar{\ell}\ell$

– $d^2\Gamma/dq^2\, d\cos\theta_l \to \Gamma,\ A_{\rm FB},\ F_H$

$B \to V + \bar{\ell}\ell$

– $d^2\Gamma/dq^2\, d\cos\theta_l \to \Gamma,\ A_{\rm FB},\ F_L$

$B \to V_{on-shell}(\to P_1 P_2) + \bar{\ell}\ell$

– $d^4\Gamma/dq^2 d\cos\theta_l d\cos\theta_{K*}\, d\phi$

$\to J^{(s,c)}_{1,\ldots,9}(q^2)$

BUT requires non-perturbative (Non-PT) input $\to$ Lattice could help

($B \to V$) FF's @ $q^2 = 0$
LCDA's

($B \to P$) FF's: $f_{+,0}$
($B \to V$) FF's: V, $A_{0,1,2}$

decay constants: f_B
($B \to P$) FF's: $f_{+,T,P}$
($B \to V$) FF's: V, $A_{0,1,2}$, $T_{1,2,3}$
LCDA's

$$B = \{B_u,\ B_d,\ B_s\}, \qquad q = \{d,\ s\}, \qquad \ell = \{e,\ \mu,\ \tau\}$$

Up to now:

"Phenomenology" searches for

A) combinations of observables to eliminate Non-PT dependences (guided by power expansions)
B) observables which test SM (suppressed or precise)
C) observables with special sensitivity to NP

Future:

"Lattice" can improve

- directly observables with Non-PT dependences
- knowledge of Non-PT dependences in sub-leading terms of power expansions (see [A)])

OUTLINE

cover here only some examples (for $b \to s$):

1) Effective theory (EFT) – $\Delta B = 1$ decays in SM and beyond
2) $B \to P + \bar{\ell}\ell$ and $B \to P + \bar{\nu}\nu$
3) $B \to V_{on-shell} (\to P_1 P_2) + \bar{\ell}\ell$
    A) kinematics and angular distribution
    B) High-q^2 analysis
4) Conclusion

GENERAL APPROACH . . .

EFT of electro-weak (EW) interactions of $\Delta B = 1$ decays in SM and beyond:

1) decoupl. of heavy degrees of freedom $(W, Z, t, \ldots)$ @ EW scale: $\mu_{EW} \gtrsim M_W$

2) RG-running to lower scale: $\mu_b \sim m_b \to$ resums large log's: $[\alpha_s \ln(\mu_b/\mu_{EW})]^n$

$$\mathcal{L}_{\rm EFT}(\mu_b) = \mathcal{L}_{\rm QED\times QCD}(u, d, s, c, b, \; e, \mu, \tau, \; ???)$$
$$+ \frac{4G_F}{\sqrt{2}} V_{\rm CKM} \sum_{\rm SM} (C_i + \Delta C_i)\mathcal{O}_i + \sum_{\rm NP} C_j \mathcal{O}_j(???)$$

Beyond the SM:

$\Rightarrow$??? . . . additional light degrees of freedom ($\Leftarrow$ not pursued)

$\Rightarrow$ ΔC_i . . . NP contributions to SM C_i

$\Rightarrow$ $\sum_{\rm NP} C_j \mathcal{O}_j(???)$ . . . NP operators (e.g. $C'_{7,9,10}$, $C^{(\prime)}_{S,P}$, . . .)

SM operator List

. . . using CKM unitarity

$$\mathcal{L}_{\rm SM} \sim \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\mathcal{L}_{\rm SM}^{(t)} + \hat{\lambda}_u \, \mathcal{L}_{\rm SM}^{(u)}\right), \qquad \hat{\lambda}_u = V_{ub} V_{us}^* / V_{tb} V_{ts}^*$$

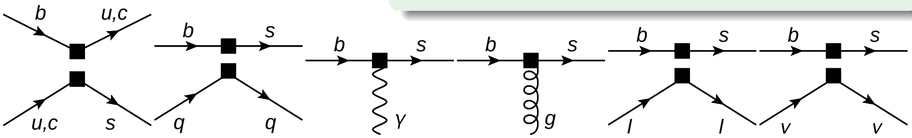
$\mathcal{L}_{\rm SM}^{(u)} = C_1(\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2(\mathcal{O}_2^c - \mathcal{O}_2^u)$

$\mathcal{O}_{1,2}^{u,c}$ = curr.-curr.: $b \to s\,\bar{u}u$, $b \to s\,\bar{c}c$

CP-violation in the SM $\to \hat{\lambda}_u$

$\mathcal{L}_{\rm SM}^{(t)} = C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i>2} C_i \mathcal{O}_i$

$\mathcal{O}_{1,2}^c$ =	curr.-curr.	$b \to s\,\bar{c}c$
$\mathcal{O}_{3,4,5,6}$ =	QCD-peng.	$b \to s\,\bar{q}q$, $q = \{u, d, s, c, b\}$
$\mathcal{O}_7^\gamma$ =	electr.magn.	$b \to s\,\gamma$
$\mathcal{O}_8^g$ =	chromo.magn.	$b \to s\,g$
$\mathcal{O}_{9,10}^{\ell\ell}, \mathcal{O}_L^{\nu\nu}$ =	semi-lept.	$b \to s\,\bar{\ell}\ell$, $b \to s\,\bar{\nu}\nu$
$\mathcal{O}_{3,4,5,6}^Q$ =	QED-peng.	$b \to s\,\bar{q}q$, $q = \{u, d, s, c, b\}$
$\mathcal{O}_b$ =	QED-box	$b \to s\,\bar{b}b$



Beyond the SM operator list

frequently considered in model-(in)dependent searches

$b \to q + \bar{\ell}\ell$

$$\mathcal{O}_S^{\ell\ell} = \frac{\alpha_e}{4\pi}[\bar{s}P_R b][\bar{\ell}\ell], \qquad \mathcal{O}_S^{\ell\ell\prime} = \frac{\alpha_e}{4\pi}[\bar{s}P_L b][\bar{\ell}\ell],$$

$$\mathcal{O}_P^{\ell\ell} = \frac{\alpha_e}{4\pi}[\bar{s}P_R b][\bar{\ell}\gamma_5\ell], \qquad \mathcal{O}_P^{\ell\ell\prime} = \frac{\alpha_e}{4\pi}[\bar{s}P_L b][\bar{\ell}\gamma_5\ell],$$

$$\mathcal{O}_T^{\ell\ell} = \frac{\alpha_e}{4\pi}[\bar{s}\sigma_{\mu\nu} b][\bar{\ell}\sigma^{\mu\nu}\ell], \qquad \mathcal{O}_{T5}^{\ell\ell} = \frac{\alpha_e}{4\pi}[\bar{s}\sigma_{\mu\nu} b][\bar{\ell}\sigma^{\mu\nu}\gamma_5\ell],$$

$b \to q + \bar{\nu}\nu$

$$\mathcal{O}_R^{\nu\nu} = \frac{\alpha_e}{4\pi}[\bar{s}\gamma^\mu P_R b][\bar{\nu}\gamma_\mu P_L \nu]$$

(neglecting ν-masses)

- new Dirac-structures beyond SM: right-handed currents, (pseudo-) scalar and/or tensor interactions
- usually added to $\mathcal{L}_{\rm SM}^{(t)}$

$\Rightarrow$ EFT starting point for calculation of observables
!!! Non-PT input required when evaluating matrix elements

$$B \to P + \bar{\ell}\ell \text{ and } B \to P + \bar{\nu}\nu$$

KINEMATICS – $B(p_B) \to P(p_P) + \bar{\ell}(p_{\bar{\ell}}) + \ell(p_\ell)$

1) $q^2 = m_{\bar{\ell}\ell}^2 = (p_{\bar{\ell}} + p_\ell)^2 = (p_B - p_P)^2$ $\qquad 4m_\ell^2 \leqslant q^2 \leqslant (M_B - M_P)^2$

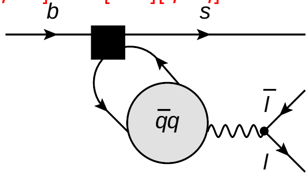
2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_B, \vec{p}_{\bar{\ell}})$ in $\bar{\ell}\ell$-c.m. system $\qquad -1 \leqslant \cos\theta_\ell \leqslant 1$

general problem in $b \to \{d, s\} + \bar{\ell}\ell$ due to Op's: $[\bar{s}\Gamma q][\bar{q}\Gamma' b]$ and $[\bar{s}\Gamma b][\bar{q}\Gamma' q]$

LONG DISTANCE - $(\bar{q}q)$-RESONANCE BACKGROUND

$$\mathcal{A}[B \to P + \bar{\ell}\ell] = \mathcal{A}[B \to P + \bar{\ell}\ell]_{SD-FCNC}$$

$$+ \mathcal{A}[B \to P + (\bar{q}q) \to P + \bar{\ell}\ell]_{LD}$$



for $B \to K + \bar{\ell}\ell$ $\qquad (q^2_{max} \approx 22.9\ \text{GeV}^2)$:

$q = u, d, s$ light resonances below $q^2 \leqslant 1\ \text{GeV}^2$
suppr. by small QCD-peng. Wilson coeff. or CKM $\hat{\lambda}_u$

$q = c$ start @ $q^2 \sim (M_{J/\psi})^2 \approx 9.6\ \text{GeV}^2$, $(M_{\psi'})^2 \approx 13.6\ \text{GeV}^2$

$\Rightarrow$ usually $\mathcal{A}[B \to P + \bar{\ell}\ell]_{SD-FCNC}$ = "non-resonant part"

q^2 - ANATOMY

K-energy E_K in B-rest frame: $2M_B E_K = M_B^2 + M_K^2 - q^2$

low-q^2	high-q^2
$q^2 \ll M_B^2$	$(M_B - M_K)^2 - 2M_B\Lambda_{\rm QCD} \lesssim q^2$
$E_K \sim M_B/2$ large recoil	$E_K \sim M_K + \Lambda_{\rm QCD}$ low recoil
$q^2 \in [1, 6]$ GeV^2 QCDF, SCET	$q^2 \geqslant 17.7(14.0)$ GeV^2 for K (K^*) OPE + HQET

low-q^2 above $q = u, d, s$ resonances and below $q = c$ resonances:
$\mathcal{A}[B \to P + (\bar{q}q) \to P + \bar{\ell}\ell]_{LD}$ treated within $(\Lambda_{\rm QCD}/m_c)^2$ expansion

high-q^2 quark-hadron duality + OPE

$\bar{B} \to \bar{K} + \bar{\ell}\ell$ MATRIX ELEMENT

$$\mathcal{M}[\bar{B} \to \bar{K}\bar{\ell}\ell] = i\frac{G_F \alpha_e}{\sqrt{2}\pi} V_{tb} V_{ts}^* \, f_+(q^2) \Bigg(F_V \, p_B^\mu \, [\bar{\ell}\gamma_\mu \ell] + F_A \, p_B^\mu \, [\bar{\ell}\gamma_\mu \gamma_5 \ell] + (F_S + \cos\theta_\ell F_T) \, [\bar{\ell}\ell] + (F_P + \cos\theta_\ell F_{T5}) \, [\bar{\ell}\gamma_5 \ell] \Bigg)$$

$$F_A = C_{10} + C'_{10}, \qquad F_V = (C_9 + C'_9) + \frac{2m_b}{M_B} \frac{\mathcal{T}_P(q^2)}{f_+(q^2)} + \frac{8 m_\ell}{(M_B + M_K)} \frac{f_T(q^2)}{f_+(q^2)} C_T^\ell,$$

$$F_P = \frac{1}{2} \frac{(M_B^2 - M_K^2)}{(m_b - m_s)} \frac{f_0(q^2)}{f_+(q^2)} (C_P^\ell + C_P^{\ell\prime}) + m_\ell (C_{10} + C'_{10}) \left[\frac{(M_B^2 - M_K^2)}{q^2} \left(\frac{f_0(q^2)}{f_+(q^2)} - 1 \right) - 1 \right],$$

$$F_S = \frac{1}{2} \frac{(M_B^2 - M_K^2)}{(m_b - m_s)} \frac{f_0(q^2)}{f_+(q^2)} (C_S^\ell + C_S^{\ell\prime}), \qquad F_{T,T5} = \frac{2\sqrt{\lambda}\,\beta_\ell}{(M_B + M_K)} \frac{f_T(q^2)}{f_+(q^2)} C_{T,T5}^\ell.$$

LOW-q^2 QCD-FACTORISATION

$B \to P$ (QCD-) FORM FACTORS $f_{+,0,T}$

$$\langle P(p_P)|\bar{s}\gamma_\mu b|B(p_B)\rangle = (2p_B - q)_\mu f_+(q^2) + \frac{M_B^2 - M_P^2}{q^2} q_\mu [f_0(q^2) - f_+(q^2)],$$

$$\langle P(p_P)|\bar{s}i\sigma_{\mu\nu}q^\nu b|B(p_B)\rangle = -[(2p_B - q)_\mu q^2 - (M_B^2 - M_K^2)q_\mu] \frac{f_T(q^2)}{M_B + M_P}.$$

QCDF FORM FACTOR (LARGE RECOIL SYMMETRY) RELATIONS

$$\frac{f_0}{f_+} = \frac{2E_K}{M_B}\left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{q^2}{M_B^2}\sqrt{\frac{\Lambda_{\rm QCD}}{E_K}}\right)\right], \quad \frac{f_T}{f_+} = \frac{M_B + M_K}{M_B}\left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\sqrt{\frac{\Lambda_{\rm QCD}}{E_K}}\right)\right]$$

Beneke/Feldmann hep-ph/0008255, Beneke/Chapovsky/Diehl/Feldmann hep-ph/0206152

Soft overlap and hard scattering contributions in $\mathcal{T}_P$ from $\mathcal{O}_7^\gamma$, $\mathcal{O}_8^g$, 4-quark ops.

$$\mathcal{T}_P^{(0)}(q^2) = f_+(q^2)\left[(C_7^{\rm eff(0)} + C_7') + \frac{M_B}{2m_b} Y^{(0)}(q^2)\right]$$

$\Leftarrow$ requires Non-PT input: B and K decay contstants and LCDA's!!!
@ NNLO QCD in Beneke/Feldmann/Seidel hep-ph/0106067 (soft overlap, hard scattering)

$\bar{B} \to \bar{K} + \bar{\ell}\ell$ OBSERVABLES

$$\frac{1}{d\Gamma_\ell/dq^2}\frac{d^2\Gamma_\ell}{dq^2\, d\cos\theta_\ell} = \frac{3}{4}[1 - F_H^\ell(q^2)](1-\cos^2\theta_\ell) + \frac{1}{2}F_H^\ell(q^2) + A_{\rm FB}^\ell(q^2)\cos\theta_\ell$$

$$R_K = \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \frac{d\Gamma[\bar{B}\to\bar{K}\bar{\mu}\mu]}{dq^2} \Big/ \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \frac{d\Gamma[\bar{B}\to\bar{K}\bar{e}e]}{dq^2}$$

SM PREDICTIONS – @ LOW-$q^2 \in [1,6]$ GeV^2

C.B./HILLER/PIRANISHVILI ARXIV:0709.4174

$F_H^\mu = 0.0243 \pm 0.0003$, $F_H^e \approx 0$, $A_{\rm FB}^\ell \approx 0$, $R_K = 1.0003 \pm 0.0001$

EXPERIMENTAL INFORMATION

$F_H^\ell = 0.81^{+0.58}_{-0.61} \pm 0.46$ (ℓ-averaged and $q^2 > 0.04$ GeV^2) [BaBar '06]

$(R_K - 1) = 0.24 \pm 0.31$ and $(0.03 \pm 0.19 \pm 0.06)$ ($q^2 > 0.04$ GeV^2) [BaBar '06 and Belle '09]

$A_{{\rm FB},\, q^2\in[1,6]\,{\rm GeV}^2} = (-0.04^{+0.13}_{-0.16} \pm 0.05)$ and $(+0.08^{+0.27}_{-0.22} \pm 0.07)$ [Belle '09 and CDF '10 $\ell = \mu$]

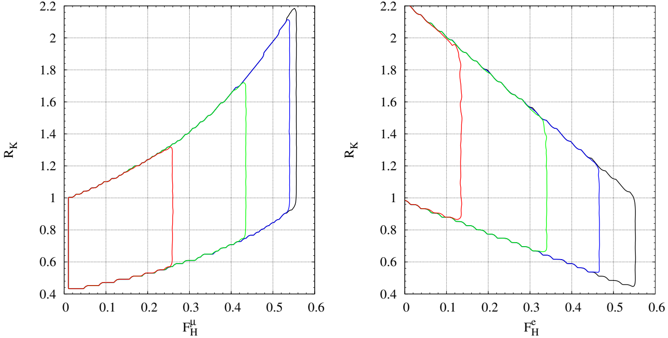
- Observables defined as ratios $\Rightarrow$ Hadronic uncertainties cancel!!!
- $F_H^\ell \sim m_\ell^2$ "(Quasi-) Nulltest" of the SM (unc. from sub-lead. terms ~ 6 %)
- R_K sensitive to non-universal lepton flavour interactions beyond SM
  (!!! should include ℓ-breaking effects from QED phase space logarithms)

incl. non-SM Op's with $\ell = \{e, \mu\}$

$$\mathcal{O}_S^{\ell\ell(')} = (\bar{s}P_{R,L}b)(\bar{\ell}\ell),$$
$$\mathcal{O}_P^{\ell\ell(')} = (\bar{s}P_{R,L}b)(\bar{\ell}\gamma_5\ell),$$
$$\Rightarrow \; C_{S,P}^{e(')},\; C_{S,P}^{\mu(')}$$

using constraints at 90% C.L. from

1) $\mathcal{B}(B_s \to \bar{e}e) < 5.4 \times 10^{-5}$
2) $\mathcal{B}(B_s \to \bar{\mu}\mu) < 8.0 \times 10^{-8}$
3) $\mathcal{B}(B \to X_s\bar{e}e)_{[q^2>0.04]} = (4.7 \pm 1.3) \times 10^{-6}$
4) $\mathcal{B}(B \to X_s\bar{\mu}\mu)_{[q^2>0.04]} = (4.3 \pm 1.2) \times 10^{-6}$



Contours of: $\mathcal{B}(B \to X_s\bar{\mu}\mu)_{[1,6]} < \{1.75, 2.0, 2.17\} \times 10^{-6}$ (left)
$\mathcal{B}(B \to X_s\bar{e}e)_{[1,6]} < \{1.75, 2.0, 2.25, 2.35\} \times 10^{-6}$ (right)

COMBINING $\bar{B} \to \bar{K} + \bar{\ell}\ell$ AND $\bar{B} \to \bar{K} + \bar{\nu}\nu$

1) $\mathcal{M}[\bar{B} \to \bar{K}\bar{\nu}\nu]$ and $\mathcal{M}[\bar{B} \to \bar{K}\bar{\ell}\ell] \sim f_+(q^2)$
2) form factor relation at high-q^2 from HQET: $f_0/f_+ = 1$ and $f_T/f_+ = (M_B + M_K)/M_B$

BARTSCH/BEYLICH/BUCHALLA/GAO ARXIV:0909.1512 PROPOSE ($s = q^2/M_B^2$)

$$R_{25} \equiv \frac{\int_0^{0.25} ds\, d\mathcal{B}[B^- \to K^-\bar{\nu}\nu]/ds}{\int_0^{0.25} ds\, d\mathcal{B}[B^- \to K^-\bar{\ell}\ell]/ds}$$

$$R_{256} \equiv \frac{\int_0^{s_m} ds\, d\mathcal{B}[B^- \to K^-\bar{\nu}\nu]/ds}{\int_0^{0.25} ds\, d\mathcal{B}[B^- \to K^-\bar{\ell}\ell]/ds + \int_{0.6}^{s_m} ds\, d\mathcal{B}[B^- \to K^-\bar{\ell}\ell]/ds}$$

with $s = 0.6 \to q^2 = 16.7$ GeV^2 and $s_m = 0.821 \to q^2 = 22.9$ GeV^2

SM PREDICTIONS

$$R_{25} = 7.60^{+0.00}_{-0.00}(a_0)^{+0.00}_{-0.00}(b_1)^{+0.36}_{-0.43}(\mu), \qquad R_{256} = 14.60^{+0.28}_{-0.38}(a_0)^{+0.10}_{-0.02}(b_1)^{+0.62}_{-0.80}(\mu)$$

a_0, b_1 form factor parametrisation, μ renormalisation scale

possible strategy: fitting q^2 form factor dependence to exp. $B^- \to K^-\bar{\ell}\ell$ spectrum, using lattice input at particular q_0^2 as normalisation $\to$ prediction for $B^- \to K^-\bar{\nu}\nu$ using $R_{25,256}$

$$B \to V_{on-shell}(\to P_1 P_2) + \bar{\ell}\ell$$

KINEMATICS AND ANGULAR DISTRIBUTION - I

KINEMATICS – $B(p_B) \to V_{on-shell}[\to P_1(p_1) + P_2(p_2)] + \bar{\ell}(p_{\bar{\ell}}) + \ell(p_\ell)$

1) $q^2 = m_{\bar{\ell}\ell}^2 = (p_{\bar{\ell}} + p_\ell)^2 = (p_B - p_P)^2$ $\qquad 4m_\ell^2 \leqslant q^2 \leqslant (M_B - M_P)^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_B, \vec{p}_{\bar{\ell}})$ in $(\bar{\ell}\ell)$-c.m. system $\qquad -1 \leqslant \cos\theta_\ell \leqslant 1$

3) $\cos\theta_{K^*}$ with $\theta_{K^*} \angle(\vec{p}_B, \vec{p}_{P_1})$ in $(P_1 P_2)$-c.m. system $\qquad -1 \leqslant \cos\theta_{K^*} \leqslant 1$

4) $\phi \angle(\vec{p}_{P_1} \times \vec{p}_{P_2}, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF $\qquad -\pi \leqslant \phi \leqslant \pi$

$$
\begin{aligned}
\frac{32\pi}{9} \frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} = {} & I_1^s \sin^2\theta_{K^*} + I_1^c \cos^2\theta_{K^*} + (I_2^s \sin^2\theta_{K^*} + I_2^c \cos^2\theta_{K^*}) \cos 2\theta_\ell \\
& + I_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi + I_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi \\
& + (I_6^s \sin^2\theta_{K^*} + I_6^c \cos^2\theta_{K^*}) \cos\theta_\ell + I_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi \\
& + I_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi + I_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi
\end{aligned}
$$

For CP-conjugated decay mode

$I_{1,2,3,4,7}^{(j)}(q^2) \to \bar{I}_{1,2,3,4,7}^{(j)}(q^2),$ $\qquad$ CP − even

$I_{5,6,8,9}^{(j)}(q^2) \to -\bar{I}_{5,6,8,9}^{(j)}(q^2),$ $\qquad$ CP − odd (= CP − asy. from untagged B samples)

Kinematics and angular distribution - II

$\Rightarrow$ 12 q^2-dependent observables $I_i^{(j)} = f_{ij}(A_a A_b^*)$ in terms of 8 transversity amplitudes

$$A^{L,R}_{\perp,\|,0} \text{ and } A_{S,t} \sim m_\ell$$

$\Rightarrow$ in the limit $m_\ell \to 0$ only 9 (U_k simple lin. comb. of $I_i^{(j)}$)

$$U_1 = |A_0^L|^2 + |A_0^R|^2, \qquad U_4 = \mathrm{Re}(A_0^L A_\|^{L*} + A_0^{R*} A_\|^R), \qquad U_7 = \mathrm{Im}(A_0^L A_\|^{L*} + A_0^{R*} A_\|^R),$$

$$U_2 = |A_\perp^L|^2 + |A_\perp^R|^2, \qquad U_5 = \mathrm{Re}(A_0^L A_\perp^{L*} - A_0^{R*} A_\perp^R), \qquad U_8 = \mathrm{Im}(A_0^L A_\perp^{L*} - A_0^{R*} A_\perp^R),$$

$$U_3 = |A_\|^L|^2 + |A_\|^R|^2, \qquad U_6 = \mathrm{Re}(A_\|^L A_\perp^{L*} - A_\|^{R*} A_\perp^R), \qquad U_9 = \mathrm{Im}(A_\|^L A_\perp^{L*} - A_\|^{R*} A_\perp^R),$$

Strategy: ratios of lin. comb. of $I_i^{(j)}$ or $h_k(A_a A_b^*)$ with reduced FF depend's or sensitivities to NP

Observables

CP-averaged	$S_i^{(j)} = (I_i^{(j)} + \bar{I}_i^{(j)}) / \frac{d(\Gamma+\bar{\Gamma})}{dq^2}$	Altmannshofer et al. arXiv:0811.1214
CP asymmetires	$A_i^{(j)} = (I_i^{(j)} - \bar{I}_i^{(j)}) / \frac{d(\Gamma+\bar{\Gamma})}{dq^2}$	
other	A_{FB}, A_{FB}^{CP}, F_L,	
	$A_T^{2,3,4}$,	Egede et al. arXiv:0807.2589, arXiv:1005.0571
	$H_T^{1,2,3}$	C.B./Hiller/van Dyk arXiv:1006.5013

7 $B \to V$ form factors $V, A_{0,1,2}, T_{1,2,3}$

$$\langle V(k,\epsilon)|\,\bar{q}\gamma_\mu b\,|B(p)\rangle = \frac{2V(q^2)}{m_B+m_V}\varepsilon_{\mu\rho\sigma\tau}\epsilon^{*\rho}p^\sigma k^\tau\,,$$

$$\langle V(k,\epsilon)|\,\bar{q}\gamma_\mu\gamma_5 b\,|B(p)\rangle = i\epsilon^{*\rho}\left[2m_V A_0(q^2)\frac{q_\mu q_\rho}{q^2} + (m_B+m_V)A_1(q^2)\left(g_{\mu\rho} - \frac{q_\mu q_\rho}{q^2}\right)\right.$$

$$\left. - A_2(q^2)\frac{q_\rho}{m_B+m_V}\left((p+k)_\mu - \frac{m_B^2-m_V^2}{q^2}(p-k)_\mu\right)\right],$$

$$\langle V(k,\epsilon)|\,\bar{q}i\sigma_{\mu\nu}q^\nu b\,|B(p)\rangle = -2T_1(q^2)\varepsilon_{\mu\rho\sigma\tau}\epsilon^{*\rho}p^\sigma k^\tau\,,$$

$$\langle V(k,\epsilon)|\,\bar{q}i\sigma_{\mu\nu}\gamma_5 q^\nu b\,|B(p)\rangle = iT_2(q^2)\left(\epsilon^*_\mu(m_B^2-m_V^2) - (\epsilon^*\cdot q)(p+k)_\mu\right)$$

$$+ iT_3(q^2)\,(\epsilon^*\cdot q)\left(q_\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right),$$

HIGH-q^2 – I

FRAMEWORK DEVELOPED IN GRINSTEIN/PIRJOL HEP-PH/0404250

1) OPE in $\Lambda_{\rm QCD}/Q$ with $Q = \{m_b, \sqrt{q^2}\}$ + matching on HQET for 4-quark Op's

$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi}{q^2} \sum_{i=1}^{6} C_i(\mu)\, \mathcal{T}_\alpha^{(i)}(q^2, \mu)\, [\bar{\ell}\gamma^\alpha \ell]$$

$$\mathcal{T}_\alpha^{(i)}(q^2, \mu) = i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{O}_i(0), j_\alpha^{\rm e.m.}(x)\} | \bar{B}\rangle$$

$$= \sum_{k \geqslant -2} \sum_j C_{i,j}^{(k)} \langle \mathcal{Q}_{j,\alpha}^{(k)} \rangle$$

(in analogy to heavy quark expansion)

$\mathcal{Q}_{j,\alpha}^{(k)}$	power	$\mathcal{O}(\alpha_s)$
$\mathcal{Q}_{1,2}^{(-2)}$	1	$\alpha_s^0(Q)$
$\mathcal{Q}_{1-5}^{(-1)}$	$\Lambda_{\rm QCD}/Q$	$\alpha_s^1(Q)$
$\mathcal{Q}_{1,2}^{(0)}$	m_c^2/Q^2	$\alpha_s^0(Q)$
$\mathcal{Q}_{j>3}^{(0)}$	${\Lambda_{\rm QCD}}^2/Q^2$	$\alpha_s^0(Q)$
$\mathcal{Q}_{j}^{(2)}$	m_c^4/Q^4	$\alpha_s^0(Q)$

inlcuded,
unc. estimate by naive pwr cont.

2) HQET FF-relations at sub-leading order

$$T_1(q^2) = \kappa\, V(q^2), \qquad T_2(q^2) = \kappa\, A_1(q^2), \qquad T_3(q^2) = \kappa\, A_2(q^2)\frac{M_B^2}{q^2},$$

$$\kappa = \left(1 + \frac{2D_0^{(v)}(\mu)}{C_0^{(v)}(\mu)}\right)\frac{m_b(\mu)}{M_B}$$

can express everything in terms of QCD FF's $V, A_{1,2}$ @ $\mathcal{O}(\alpha_s \Lambda_{\rm QCD}/Q)$!!!

HIGH-q^2 – II

TRANSVERSITY AMPLITUDES - SM OP'S ONLY

$$A_\perp^{L,R} = +iNM_B\, c_{L,R}\, f_\perp, \qquad A_\parallel^{L,R} = -iNM_B\, c_{L,R}\, f_\parallel, \qquad A_0^{L,R} = -iNM_B\, c_{L,R}\, f_0$$

universal "short-distance" coefficient

$$c_{L,R} = (\mathcal{C}_9^{eff} \mp \mathcal{C}_{10}) + \kappa \frac{2\hat{m}_b}{\hat{s}}\, \mathcal{C}_7^{eff}$$

Non-PT FF's ("helicity FF's" Bharucha/Feldmann/Wick arXiv:1004.3249)

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K^*}} V, \qquad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K^*})\, A_1, \qquad f_0 = \frac{(1-\hat{s}-\hat{M}_{K^*}^2)(1+\hat{M}_{K^*})^2 A_1 - \hat{\lambda}\, A_2}{2\,\hat{M}_{K^*}\,(1+\hat{M}_{K^*})\sqrt{\hat{s}}}$$

OBSERVABLES $\sim A_i A_j^* \sim U_k \sim \rho_{1,2}$

$$\rho_1 \equiv \left|\mathcal{C}_9^{eff} + \kappa \frac{2\hat{m}_b}{\hat{s}} \mathcal{C}_7^{eff}\right|^2 + |\mathcal{C}_{10}|^2, \qquad \rho_2 \equiv \mathrm{Re}\left(\mathcal{C}_9^{eff} + \kappa \frac{2\hat{m}_b}{\hat{s}} \mathcal{C}_7^{eff}\right) \mathcal{C}_{10}^*$$

$$U_1 = 2\rho_1 f_0^2, \qquad U_2 = 2\rho_1 f_\perp^2, \qquad U_3 = 2\rho_1 f_\parallel^2,$$

$$U_4 = 2\rho_1 f_0 f_\parallel, \qquad U_5 = 4\rho_2 f_0 f_\perp, \qquad U_6 = 4\rho_2 f_\parallel f_\perp, \qquad U_7 = U_8 = U_9 = 0.$$

ρ_1 and ρ_2 are largely μ-scale independent

High-q^2 – III

FF-free ratios !!! test SD flavour couplings versus exp. data + OPE

$$H_T^{(1)} = \frac{U_4}{\sqrt{U_1 \cdot U_3}} = 1, \qquad H_T^{(2)} = \frac{U_5}{\sqrt{U_1 \cdot U_2}} = 2\,\frac{\rho_2}{\rho_1}, \qquad H_T^{(3)} = \frac{U_6}{\sqrt{U_2 \cdot U_3}} = 2\,\frac{\rho_2}{\rho_1},$$

integr. results in $q^2_{\rm min} = 14\,\mathrm{GeV}^2 < q^2 \leq 19.2\,\mathrm{GeV}^2 = q^2_{\rm max}$, based on extrapolated LCSR FF's (Ball/Zwicky hep-ph/0412079) assuming same uncertainties – Lattice results desireable

$$10^7 \cdot \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2\, \frac{d\mathcal{B}}{dq^2} = 2.96\, {}^{+0.90}_{-0.77}\Big|_{\rm FF}\, {}^{+0.18}_{-0.17}\Big|_{\rm SL} \pm 0.10\Big|_{\rm IWR} \pm 0.16\Big|_{\rm CKM}\, {}^{+0.08}_{-0.06}\Big|_{\rm SD}$$

$$\langle A_{\rm FB}\rangle = -0.41 \pm 0.07\Big|_{\rm FF} \pm 0.02\Big|_{\rm SL} \pm 0.01\Big|_{\rm IWR}\, {}^{+0.001}_{-0.002}\Big|_{\rm SD},$$

$$\langle F_{\rm L}\rangle = 0.35\, {}^{+0.04}_{-0.05}\Big|_{\rm FF} \pm 0.03\Big|_{\rm SL} \pm 0.02\Big|_{\rm IWR},$$

$$\langle H_T^{(1)}\rangle = +0.997 \pm 0.002\Big|_{\rm FF}\, {}^{+0.000}_{-0.001}\Big|_{\rm IWR},$$

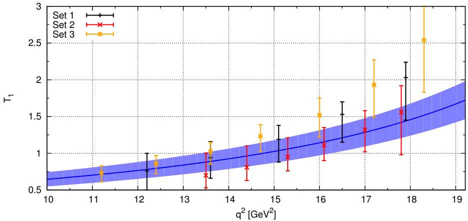
$$\langle H_T^{(2)}\rangle = -0.972\, {}^{+0.004}_{-0.003}\Big|_{\rm FF} \pm 0.001\Big|_{\rm SL}\, {}^{+0.008}_{-0.005}\Big|_{\rm IWR}\, {}^{+0.003}_{-0.004}\Big|_{\rm SD},$$

$$\langle H_T^{(3)}\rangle = -0.958 \pm 0.001\Big|_{\rm SL}\, {}^{+0.008}_{-0.006}\Big|_{\rm IWR}\, {}^{+0.003}_{-0.004}\Big|_{\rm SD}$$

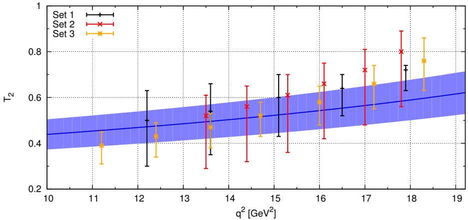
HIGH-q^2 – IV

SHORT-DISTANCE-FREE RATIOS !!! TEST LATTICE VERSUS EXP. DATA + OPE

$$\frac{f_0}{f_\parallel} = \sqrt{\frac{U_1}{U_3}} = \frac{U_1}{U_4} = \frac{U_4}{U_3} = \frac{U_5}{U_6}, \qquad \frac{f_0}{f_\perp} = \sqrt{\frac{U_1}{U_2}}, \qquad \frac{f_\perp}{f_\parallel} = \sqrt{\frac{U_2}{U_3}} = \frac{\sqrt{U_1 U_2}}{U_4}$$



LCSR extrapolation (Ball/Zwicky hep-ph/0412079) of $T_1(q^2)$ and $T_2(q^2)$ to high-q^2 versus quenched Lattice (3 data sets from Becirevic/Lubicz/Mescia hep-ph/0611295)



new results to come →
Liu/Meinel/Hart/Horgan/Müller/Wingate arXiv:0911.2370

CONCLUSION

- rich phenomenology in $b \to q + \{\gamma, \bar{\nu}\nu, \bar{\ell}\ell\}$ to test flavour short-distance couplings
- low-q^2 and high-q^2 regions in $b \to s + \bar{\ell}\ell$ accesible via power exp's
(QCDF, SCET, OPE + HQET) $\longrightarrow$ reveal symmetries of QCD dynamics
- reducing Non-PT uncertainties by suiteable ratios of observables guided by power exp's
$\longrightarrow$ allowing for quite precise theory predictions
- STILL Non-PT input from Lattice can improve theoretical predictions
$\longrightarrow$ required to exploit exp. data $d\mathcal{B}/dq^2$
- angular analysis of $B \to V_{on-shell}(\to P_1 P_2) + \bar{\ell}\ell$ @ high-q^2 offers observables to test FF (ratios) q^2-dependence independently of SD-couplings

Lattice results would be very welcome for decay constants, FF's and possibly other Non-PT input (e.g. distribution functions)!!!

Literature - incomplete

- $b \to q + \bar{\ell}\ell$ in QCDF @ low-q^2: Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400
- $b \to s + \bar{\ell}\ell$ in SCET @ low-q^2: Ali/Kramer/Zhu hep-ph/0601034
- $b \to s + \bar{\ell}\ell$ in OPE + HQET @ high-q^2: Grinstein/Pirjol hep-ph/0404250
- $b \to s + \bar{\ell}\ell$ and $\bar{c}c$-resonances:
  Buchalla/Isidori/Rey hep-ph/9705253
  Beneke/Buchalla/Neubert/Sachrajda arXiv:0902.4446
  Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945

- $B \to \{V, P\} + \bar{\nu}\nu$
  Altmannshofer/Buras/Straub/Wick arXiv:0902.0160
  Bartsch/Beylich/Buchalla/Gao arXiv:0909.1512
- $B \to P + \bar{\ell}\ell$
  Bobeth/Hiller/Piranishvili arXiv:0709.4174 : angular analysis @ low-q^2 + (pseudo-) scalar & tensor Op's
  Alok/Dighe/Uma-Sankar arXiv:0810.3779
- $B \to V_{on-shell}(\to P_1 P_2) + \bar{\ell}\ell$
  Krüger/Sehgal/Sinha/Sinha hep-ph/9907386 : CP asymmetries @ all-q^2
  Feldmann/Matias hep-ph/0212158 : isospin asymmetry A_I @ low-q^2
  Krüger/Matias hep-ph/0502060 : transv. observables @ low-q^2
  Kim/Yoshikawa arXiv:0711.3880 : @ all-q^2, also $B \to S_{on-shell}(\to P_1 P_2) + \bar{\ell}\ell$
  Bobeth/Hiller/Piranishvili arXiv:0805.2525 : CP asymmetries @ low-q^2
  Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 : LHCb and transv. observables @ low-q^2
  Altmannshofer/Ball/Bharucha/Buras/Straub/Wick arXiv:0811.1214 : CP-ave + asy @ low-q^2 + (pseudo-) scalar Op's
  Alok/Dighe/Ghosh/London/Matias/Nagashima/Szynkman arXiv:0912.1382
  Bharucha/Reece arXiv:1002.4310 : early LHCb potential @ low-q^2
  Egede/Hurth/Matias/Ramon/Reece arXiv:1005.0571 : LHCb and transv. observables@ low-q^2
  Bobeth/Hiller/van Dyk arXiv:1006.5013 : @ high-q^2
  Alok et al. arXiv:1008.2367 : @ all-q^2 + tensor Op's