Model-independent fits of $b \to s\ell^+\ell^-$ and $b \to s\gamma$ rare decays

Christoph Bobeth
TU Munich – IAS

in collaboration with
Frederik Beaujean and Danny van Dyk

ERC-Workshop
Munich

Outline

I) Introduction

- EFT of $|\Delta B| = |\Delta S| = 1$ decays
- Measurements
- Optimized Observables

II) Model-independent Fits

– Introduction –

B-Hadron decays are a Multi-scale problem . . .

. . . with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in B restframe	$\gg$	QCD-bound state effects
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV		$M_B \approx 5$ GeV		$\Lambda_{\mathrm{QCD}} \approx 0.5$ GeV

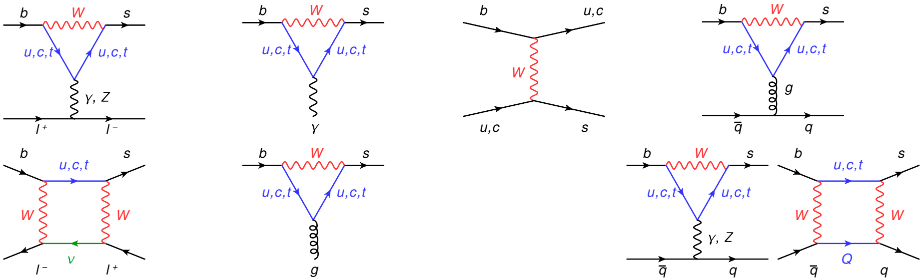
B-Hadron decays are a Multi-scale problem ...

... with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in B restframe
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV		$M_B \approx 5$ GeV

$$\mathcal{L}_{\rm eff} \sim G_F\, V_{\rm CKM} \times \Big[\sum_{9,10} C_i^{\ell\bar\ell}\, \mathcal{O}_i^{\ell\bar\ell} + \sum_{7\gamma,\, 8g} C_i\, \mathcal{O}_i + \mathrm{CC} + (\text{QCD \& QED-peng}) \Big]$$

semi-leptonic | electro- & chromo-mgn | charged current | QCD & QED -penguin



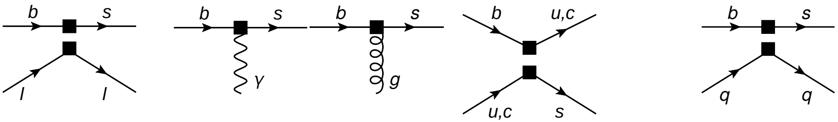
B-Hadron decays are a Multi-scale problem . . .

. . . with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in B restframe
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV		$M_B \approx 5$ GeV

$$\mathcal{L}_{\text{eff}} \sim G_F\, V_{\text{CKM}} \times \left[\sum_{9,10} C_i^{\ell\bar\ell}\, \mathcal{O}_i^{\ell\bar\ell} + \sum_{7\gamma,\,8g} C_i\, \mathcal{O}_i + \text{CC} + (\text{QCD \& QED-peng}) \right]$$

semi-leptonic | electro- & chromo-mgn | charged current | QCD & QED -penguin



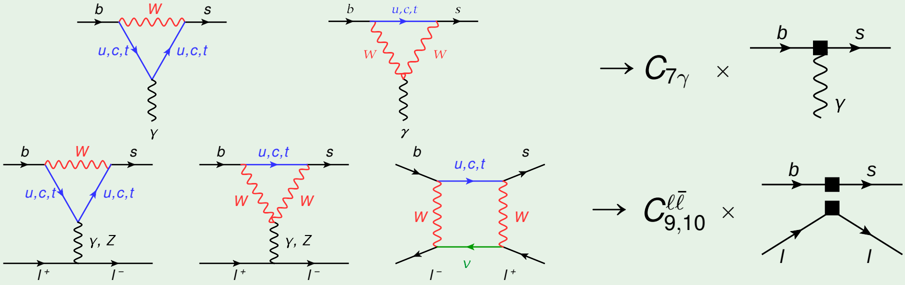
$C_i =$ **Wilson coefficients:** contains short-dist. pmr's (heavy masses $M_t, \ldots$ – CKM factored out) and leading logarithmic QCD-corrections to all orders in α_s

$\Rightarrow$ in SM known up to next-to-next-to-leading order

$\mathcal{O}_i =$ **higher-dim. operators:** flavour-changing coupling of light quarks

EFT (Effective Field Theory) in the SM (Standard Model) for . . .

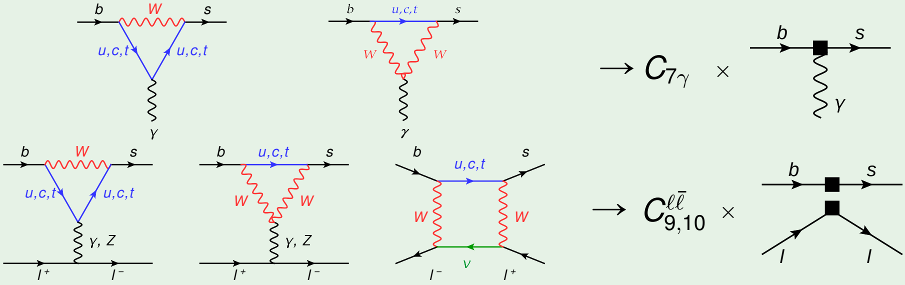
$b \to s + \gamma$ and $b \to s + \ell^+\ell^-$



$$\mathcal{O}_{7\gamma} = \frac{e}{(4\pi)^2} m_b [\bar{s}\, \sigma^{\mu\nu} P_R\, b] F_{\mu\nu}, \qquad \mathcal{O}_{9,10}^{\ell\bar{\ell}} = \frac{\alpha_e}{4\pi} [\bar{s}\, \gamma^\mu P_L\, b][\bar{\ell}\, \gamma_\mu (1,\, \gamma_5)\, \ell]$$

EFT (Effective Field Theory) in the SM (Standard Model) for . . .

$b \to s + \gamma$ and $b \to s + \ell^+\ell^-$



$$\mathcal{O}_{7\gamma} = \frac{e}{(4\pi)^2} m_b [\bar{s}\,\sigma^{\mu\nu} P_R\, b] F_{\mu\nu}, \qquad \mathcal{O}_{9,10}^{\ell\bar{\ell}} = \frac{\alpha_e}{4\pi} [\bar{s}\,\gamma^\mu P_L\, b][\bar{\ell}\,\gamma_\mu (1,\, \gamma_5)\,\ell]$$

and

- current-current op's $b \to s + Q\overline{Q}$, $(Q = u, c)$
- QCD penguin op's $b \to s + q\bar{q}$, $(q = u, d, s, c, b)$
- chromo-magnetic dipole $b \to s + gluon$

⇒ induce backgrounds

$$b \to s + (q\bar{q}) \to s + \ell^+\ell^-$$

vetoed in exp's for $q = c$: J/ψ and ψ'

Extension of EFT beyond the SM . . .

$$\mathcal{L}_{\rm eff}(\mu_b) = \mathcal{L}_{\rm QED\times QCD}(u, d, s, c, b,\ e, \mu, \tau,\ ???)$$

$$+ \frac{4G_F}{\sqrt{2}} V_{\rm CKM} \sum_{\rm SM} (C_i + \Delta C_i)\mathcal{O}_i + \sum_{\rm NP} C_j \mathcal{O}_j(???)$$

- $\Rightarrow$ ΔC_i . . . NP contributions to SM C_i
- $\Rightarrow$ $\sum_{\rm NP} C_j \mathcal{O}_j$ . . . NP operators (e.g. $C'_{7,9,10}$, $C^{(\prime)}_{S,P}$, . . .)
- $\Rightarrow$??? . . . additional light degrees of freedom ($\Leftarrow$ not pursued in the following)

Extension of EFT beyond the SM . . .

$$\mathcal{L}_{\rm eff}(\mu_b) = \mathcal{L}_{\rm QED\times QCD}(u, d, s, c, b,\ e, \mu, \tau,\ ???)$$

$$+ \frac{4G_F}{\sqrt{2}} V_{\rm CKM} \sum_{\rm SM} (C_i + \Delta C_i)\mathcal{O}_i + \sum_{\rm NP} C_j \mathcal{O}_j(???)$$

- $\Rightarrow$ ΔC_i . . . NP contributions to SM C_i
- $\Rightarrow$ $\sum_{\rm NP} C_j \mathcal{O}_j$ . . . NP operators (e.g. $C'_{7,9,10}$, $C^{(\prime)}_{S,P}$, . . .)
- $\Rightarrow$??? . . . additional light degrees of freedom ($\Leftarrow$ not pursued in the following)

model-dep. 1) decoupling of new heavy particles @ NP scale: $\mu_{NP} \gtrsim M_W$

2) RG-running to lower scale $\mu_b \sim m_b$ (potentially tower of EFT's)

C_i are correlated $\Rightarrow$ depend on fundamental parameters

model-indep. extending SM EFT-Lagrangian $\rightarrow$ new C_j

C_j are UN-correlated free parameters

Extension of EFT beyond the SM ...

$$\mathcal{L}_{\text{eff}}(\mu_b) = \mathcal{L}_{\text{QED}\times\text{QCD}}(u, d, s, c, b, e, \mu, \tau, ???)$$

$$+ \frac{4G_F}{\sqrt{2}} V_{\text{CKM}} \sum_{\text{SM}} (C_i + \Delta C_i)\mathcal{O}_i + \sum_{\text{NP}} C_j \mathcal{O}_j(???)$$

$\Rightarrow$ ΔC_i ... NP contributions to SM C_i

$\Rightarrow$ $\sum_{\text{NP}} C_j \mathcal{O}_j$ ... NP operators (e.g. $C'_{7,9,10}$, $C^{(\prime)}_{S,P}$, ...)

$\Rightarrow$??? ... additional light degrees of freedom ($\Leftarrow$ not pursued in the following)

model-dep. 1) decoupling of new heavy particles @ NP scale: $\mu_{NP} \gtrsim M_W$

2) RG-running to lower scale $\mu_b \sim m_b$ (potentially tower of EFT's)

C_i are correlated $\Rightarrow$ depend on fundamental parameters

model-indep. extending SM EFT-Lagrangian $\rightarrow$ new C_j

C_j are UN-correlated free parameters

More $b \to s + (\gamma,\, \ell^+\ell^-)$ operators beyond the SM ...

... frequently considered in model-(in)dependent searches

SM' = χ-flipped SM analogues

$$\mathcal{O}_{7'\gamma} = \frac{e}{(4\pi)^2} m_b [\bar{s}\, \sigma_{\mu\nu} P_L\, b] F^{\mu\nu}, \qquad \mathcal{O}^{\ell\bar{\ell}}_{9',10'} = \frac{\alpha_e}{4\pi} [\bar{s}\, \gamma^\mu P_R\, b][\bar{\ell}\, \gamma^\mu (1,\, \gamma_5)\, \ell]$$

S + P = scalar + pseudoscalar

$$\mathcal{O}^{\ell\bar{\ell}}_{S,S'} = \frac{\alpha_e}{4\pi} [\bar{s}\, P_{R,L}\, b][\bar{\ell}\, \ell], \qquad \mathcal{O}^{\ell\bar{\ell}}_{P,P'} = \frac{\alpha_e}{4\pi} [\bar{s}\, P_{R,L}\, b][\bar{\ell}\, \gamma_5\, \ell]$$

T + T5 = tensor

$$\mathcal{O}^{\ell\bar{\ell}}_{T} = \frac{\alpha_e}{4\pi} [\bar{s}\, \sigma_{\mu\nu}\, b][\bar{\ell}\, \sigma^{\mu\nu}\, \ell], \qquad \mathcal{O}^{\ell\bar{\ell}}_{T5} = \frac{\alpha_e}{4\pi}\, i\, \frac{\varepsilon^{\mu\nu\alpha\beta}}{2} [\bar{s}\, \sigma_{\mu\nu}\, b][\bar{\ell}\, \sigma_{\alpha\beta}\, \ell]$$

new Dirac-structures beyond SM:

- SM' : right-handed currents
- S + P : higgs-exchange & box-type diagrams
- T + T5 : box-type diagrams, Fierzed scalar tree exchange

More $b \to s + (\gamma, \ell^+\ell^-)$ operators beyond the SM . . .

. . . frequently considered in model-(in)dependent searches

SM' = χ-flipped SM analogues

$$\mathcal{O}_{7'\gamma} = \frac{e}{(4\pi)^2} m_b [\bar{s}\,\sigma_{\mu\nu} P_L\, b] F^{\mu\nu}, \qquad \mathcal{O}^{\ell\bar{\ell}}_{9',10'} = \frac{\alpha_e}{4\pi} [\bar{s}\,\gamma^\mu P_R\, b][\bar{\ell}\,\gamma^\mu (1,\, \gamma_5)\, \ell]$$

S + P = scalar + pseudoscalar

$$\mathcal{O}^{\ell\bar{\ell}}_{S,S'} = \frac{\alpha_e}{4\pi} [\bar{s}\, P_{R,L}\, b][\bar{\ell}\,\ell], \qquad \mathcal{O}^{\ell\bar{\ell}}_{P,P'} = \frac{\alpha_e}{4\pi} [\bar{s}\, P_{R,L}\, b][\bar{\ell}\,\gamma_5\, \ell]$$

T + T5 = tensor

$$\mathcal{O}^{\ell\bar{\ell}}_{T} = \frac{\alpha_e}{4\pi} [\bar{s}\,\sigma_{\mu\nu}\, b][\bar{\ell}\,\sigma^{\mu\nu}\, \ell], \qquad \mathcal{O}^{\ell\bar{\ell}}_{T5} = \frac{\alpha_e}{4\pi}\, i\, \frac{\varepsilon^{\mu\nu\alpha\beta}}{2} [\bar{s}\,\sigma_{\mu\nu}\, b][\bar{\ell}\,\sigma_{\alpha\beta}\, \ell]$$

new Dirac-structures beyond SM:

- SM' : right-handed currents
- S + P : higgs-exchange & box-type diagrams
- T + T5 : box-type diagrams, Fierzed scalar tree exchange

More $b \to s + (\gamma,\, \ell^+\ell^-)$ operators beyond the SM ...

... frequently considered in model-(in)dependent searches

SM' = χ-flipped SM analogues

$$\mathcal{O}_{7'\gamma} = \frac{e}{(4\pi)^2} m_b [\bar{s}\, \sigma_{\mu\nu} P_L\, b] F^{\mu\nu}, \qquad \mathcal{O}^{\ell\bar{\ell}}_{9',10'} = \frac{\alpha_e}{4\pi} [\bar{s}\, \gamma^\mu P_R\, b][\bar{\ell}\, \gamma^\mu (1,\, \gamma_5)\, \ell]$$

S + P = scalar + pseudoscalar

$$\mathcal{O}^{\ell\bar{\ell}}_{S,S'} = \frac{\alpha_e}{4\pi} [\bar{s}\, P_{R,L}\, b][\bar{\ell}\, \ell], \qquad \mathcal{O}^{\ell\bar{\ell}}_{P,P'} = \frac{\alpha_e}{4\pi} [\bar{s}\, P_{R,L}\, b][\bar{\ell}\, \gamma_5\, \ell]$$

T + T5 = tensor

$$\mathcal{O}^{\ell\bar{\ell}}_{T} = \frac{\alpha_e}{4\pi} [\bar{s}\, \sigma_{\mu\nu}\, b][\bar{\ell}\, \sigma^{\mu\nu}\, \ell], \qquad \mathcal{O}^{\ell\bar{\ell}}_{T5} = \frac{\alpha_e}{4\pi}\, i\, \frac{\varepsilon^{\mu\nu\alpha\beta}}{2} [\bar{s}\, \sigma_{\mu\nu}\, b][\bar{\ell}\, \sigma_{\alpha\beta}\, \ell]$$

new Dirac-structures beyond SM:

- SM' : right-handed currents
- S + P : higgs-exchange & box-type diagrams
- T + T5 : box-type diagrams, Fierzed scalar tree exchange

Experimental data: $b \to s \ell^+ \ell^-$ – number of events

# of evts	BaBar 2012 471 M $\bar{B}B$	Belle 2009 605 fb^{-1}	CDF 2011 9.6 fb^{-1}	LHCb 2011 (+2012) 1 (+2) fb^{-1}	CMS 2011 (+2012) 5 (+20) fb^{-1}	ATLAS 2011 5 fb^{-1}
$B^0 \to K^{*0} \ell\bar{\ell}$	$137 \pm 44^\dagger$	$247 \pm 54^\dagger$	288 ± 20	900 ± 34	415 ± 70	426 ± 94
$B^+ \to K^{*+} \ell\bar{\ell}$			24 ± 6	76 ± 16		
$B^+ \to K^+ \ell\bar{\ell}$	$153 \pm 41^\dagger$	$162 \pm 38^\dagger$	319 ± 23	1232 ± 40 1830 @ hi-q^2		
$B^0 \to K_S^0 \ell\bar{\ell}$			32 ± 8	60 ± 19		
$B_s \to \phi \ell\bar{\ell}$			62 ± 9	174 ± 15		
$B_s \to \mu\bar{\mu}$				emerging	emerging	limit
$\Lambda_b \to \Lambda \ell\bar{\ell}$			51 ± 7	78 ± 12		
$B^+ \to \pi^+ \ell\bar{\ell}$		limit		25 ± 7		
$B_d \to \mu\bar{\mu}$			limit	limit	limit	limit

- CP-averaged results
- J/ψ and ψ' q^2-regions vetoed
- † unknown mixture of B^0 and $B^\pm$

Babar arXiv:1204.3933 + 1205.2201
Belle arXiv:0904.0770
CDF arXiv:1107.3753 + 1108.0695 + Public Note 10894
LHCb arXiv:1205.3422 + 1209.4284 + 1210.2645 + 1210.4492 + 1304.6325 + 1305.2168 + 1306.2577 + 1307.5024 + 1307.7595 + 1308.1340 + 1308.1707
CMS arXiv:1307.5025 + 1308.3409
ATLAS ATLAS-CONF-2013-038

Experimental data: $b \to s\,\ell^+\ell^-$ – number of events

# of evts	BaBar 2012 471 M $\bar{B}B$	Belle 2009 605 fb^{-1}	CDF 2011 9.6 fb^{-1}	LHCb 2011 (+2012) 1 $(+2)$ fb^{-1}	CMS 2011 (+2012) 5 $(+20)$ fb^{-1}	ATLAS 2011 5 fb^{-1}
$B^0 \to K^{*0}\,\ell\bar{\ell}$	$137 \pm 44^\dagger$	$247 \pm 54^\dagger$	288 ± 20	900 ± 34	415 ± 70	426 ± 94
$B^+ \to K^{*+}\ell\bar{\ell}$			24 ± 6	76 ± 16		
$B^+ \to K^+\ell\bar{\ell}$	$153 \pm 41^\dagger$	$162 \pm 38^\dagger$	319 ± 23	1232 ± 40 1830 @ hi-q^2		
$B^0 \to K_S^0\,\ell\bar{\ell}$			32 ± 8	60 ± 19		
$B_s \to \phi\,\ell\bar{\ell}$			62 ± 9	174 ± 15		
$B_s \to \mu\bar{\mu}$				emerging	emerging	limit
$\Lambda_b \to \Lambda\,\ell\bar{\ell}$			51 ± 7	78 ± 12		
$B^+ \to \pi^+\,\ell\bar{\ell}$		limit		25 ± 7		
$B_d \to \mu\bar{\mu}$			limit	limit	limit	limit

Outlook / Prospects

- **Belle** reprocessed all data 711 fb^{-1} $\to$ final analysis ?
- **LHCb** $\sim$ 2 fb^{-1} from 2012 to be analysed and $(5-7)$ fb^{-1} by the end of 2017
- **ATLAS / CMS** $\sim$ 20 fb^{-1} from 2012 to be analysed
- **Belle II** expects about (10-15) K events $B \to K^*\ell\bar{\ell}$ ($\gtrsim$ 2020) [Bevan arXiv:1110.3901]

Angular analysis of $\overline{B} \to \overline{K}^* [\to \overline{K}\pi] + \ell^+\ell^-$

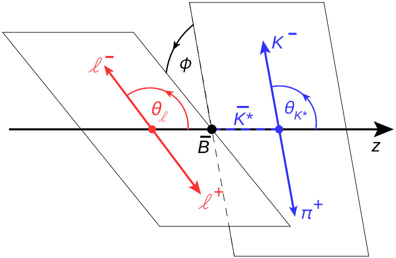
4-body decay with on-shell $\overline{K}^*$ (vector)

1) $q^2 = m^2_{\ell\bar{\ell}} = (p_\ell + p_{\bar{\ell}})^2 = (p_{\bar{B}} - p_{\bar{K}*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_{\bar{B}}, \vec{p}_\ell)$ in $(\ell\bar{\ell})$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle(\vec{p}_{\bar{B}}, \vec{p}_{\bar{K}})$ in $(\bar{K}\pi)$ – c.m. system

4) ϕ $\angle(\vec{p}_{\bar{K}} \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



Angular analysis of $\overline{B} \to \overline{K}^* [\to \overline{K}\pi] + \ell^+\ell^-$

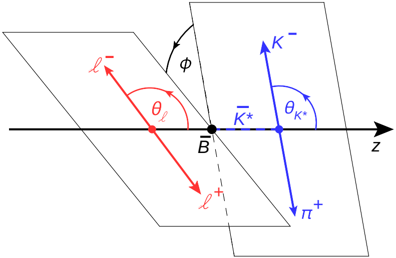
4-body decay with on-shell $\overline{K}^*$ (vector)

1) $q^2 = m^2_{\ell\bar\ell} = (p_\ell + p_{\bar\ell})^2 = (p_{\bar B} - p_{\bar K *})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_{\bar B}, \vec{p}_\ell)$ in $(\ell\bar\ell)$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle(\vec{p}_{\bar B}, \vec{p}_{\bar K})$ in $(\bar K\pi)$ – c.m. system

4) $\phi\ \angle(\vec{p}_{\bar K} \times \vec{p}_\pi, \vec{p}_{\bar\ell} \times \vec{p}_\ell)$ in B-RF



$J_i(q^2)$ = "Angular Observables"

$$\frac{32\pi}{9}\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_\ell$$

$$+J_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + J_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$

$$+(J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_\ell + J_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$

$$+J_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + J_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi$$

Angular analysis of $\overline{B} \to \overline{K}^* [\to \overline{K}\pi] + \ell^+\ell^-$

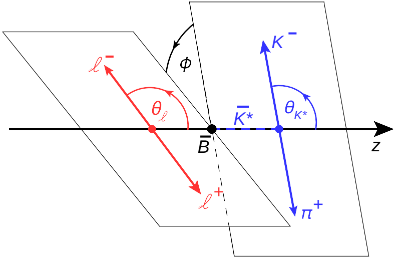
4-body decay with on-shell $\overline{K}^*$ (vector)

1) $q^2 = m^2_{\ell\bar{\ell}} = (p_\ell + p_{\bar{\ell}})^2 = (p_{\bar{B}} - p_{\bar{K}*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_{\bar{B}}, \vec{p}_\ell)$ in $(\ell\bar{\ell})$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle(\vec{p}_{\bar{B}}, \vec{p}_{\bar{K}})$ in $(\bar{K}\pi)$ – c.m. system

4) $\phi \angle(\vec{p}_{\bar{K}} \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



$J_i(q^2)$ = "Angular Observables"

$$\frac{32\pi}{9} \frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K + (J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_\ell$$
$$+ J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$
$$+ (J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi$$
$$+ J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$

$\Rightarrow$ "$2 \times (12 + 12) = 48$" if measured separately: A) decay + CP-conj and B) for $\ell = e, \mu$

Angular analysis of $\overline{B} \to \overline{K}^* [\to \overline{K}\pi] + \ell^+\ell^-$

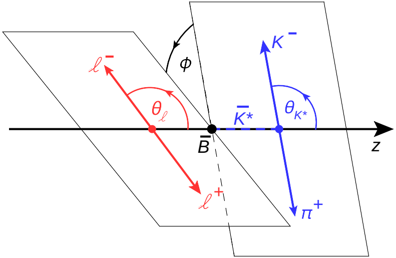
4-body decay with on-shell $\overline{K}^*$ (vector)

1) $q^2 = m^2_{\ell\bar{\ell}} = (p_\ell + p_{\bar{\ell}})^2 = (p_{\bar{B}} - p_{\bar{K}*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_{\bar{B}}, \vec{p}_\ell)$ in $(\ell\bar{\ell})$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle(\vec{p}_{\bar{B}}, \vec{p}_{\bar{K}})$ in $(\bar{K}\pi)$ – c.m. system

4) $\phi \angle(\vec{p}_{\bar{K}} \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



$\Rightarrow$ CP-averaged and CP-asymmetric angular observables

$$S_i = \frac{J_i + \bar{J}_i}{\Gamma + \bar{\Gamma}}, \qquad A_i = \frac{J_i - \bar{J}_i}{\Gamma + \bar{\Gamma}},$$

[Krüger/Sehgal/Sinha/Sinha hep-ph/9907386]
[Altmannshofer et al. arXiv:0811.1214]

CP-conj. decay $B^0 \to K^{*0}(\to K^+\pi^-)\ell^+\ell^-$: $d^4\bar{\Gamma}$ from $d^4\Gamma$ by replacing

CP-even	:	$J_{1,2,3,4,7}$	$\longrightarrow$	$+\bar{J}_{1,2,3,4,7}[\delta_W \to -\delta_W]$
CP-odd	:	$J_{5,6,8,9}$	$\longrightarrow$	$-\bar{J}_{5,6,8,9}[\delta_W \to -\delta_W]$

with weak phases δ_W conjugated

"Optimized observables" in $B \to K^* \ell^+ \ell^-$

Idea: reduce form factor (FF) sensitivity by combination (usually ratios) of angular obs's J_i

$\Rightarrow$ guided by large energy limit @ low-q^2 and Isgur-Wise @ high-q^2 FF-relations

“Optimized observables” in $B \to K^* \ell^+ \ell^-$

Idea: reduce form factor (FF) sensitivity by combination (usually ratios) of angular obs's J_i

$\Rightarrow$ guided by large energy limit @ low-q^2 and Isgur-Wise @ high-q^2 FF-relations

@ low q^2 = large recoil

$$A_T^{(2)} = P_1 = \frac{J_3}{2\,J_{2s}}, \qquad A_T^{(\mathrm{re})} = 2\,P_2 = \frac{J_{6s}}{4\,J_{2s}}, \qquad A_T^{(\mathrm{im})} = -2\,P_3 = \frac{J_9}{2\,J_{2s}},$$

$$P_4' = \frac{J_4}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_5' = \frac{J_5/2}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_6' = \frac{-J_7/2}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_8' = \frac{-J_8}{\sqrt{-J_{2c}J_{2s}}},$$

$$A_T^{(3)} = \sqrt{\frac{(2\,J_4)^2 + J_7^2}{-2\,J_{2c}\,(2\,J_{2s} + J_3)}}, \qquad A_T^{(4)} = \sqrt{\frac{J_5^2 + (2\,J_8)^2}{(2\,J_4)^2 + J_7^2}}$$

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571]

[Becirevic/Schneider arXiv:1106.3283]

[Matias/Mescia/Ramon/Virto arXiv:1202.4266]

[Descotes-Genon/Matias/Ramon/Virto arXiv:1207.2753]

"Optimized observables" in $B \to K^* \ell^+ \ell^-$

Idea: reduce form factor (FF) sensitivity by combination (usually ratios) of angular obs's J_i

$\Rightarrow$ guided by large energy limit @ low-q^2 and Isgur-Wise @ high-q^2 FF-relations

@ high q^2 = low recoil

$$H_T^{(1)} = P_4 = \frac{\sqrt{2} J_4}{\sqrt{-J_{2c}(2J_{2s} - J_3)}},$$

$$H_T^{(2)} = P_5 = \frac{J_5/\sqrt{2}}{\sqrt{-J_{2c}(2J_{2s} + J_3)}}, \qquad H_T^{(3)} = \frac{J_{6s}/2}{\sqrt{(2J_{2s})^2 - (J_3)^2}},$$

$$H_T^{(4)} = Q = \frac{\sqrt{2} J_8}{\sqrt{-J_{2c}(2J_{2s} + J_3)}}, \qquad H_T^{(5)} = \frac{-J_9}{\sqrt{(2J_{2s})^2 - (J_3)^2}},$$

$$\frac{A_9}{A_{\rm FB}} = \frac{J_9}{J_{6s}}, \qquad \text{and} \qquad \frac{J_8}{J_5}$$

[CB/Hiller/van Dyk arXiv:1006.5013]
[Matias/Mescia/Ramon/Virto arXiv:1202.4266]
[CB/Hiller/van Dyk arXiv:1212.2321]

– Model-independent Fits –

"Global Fit" = combination of $b \to s + (\gamma, \ell^+\ell^-)$ observables

Parameters of interest

$$\vec{\theta} = (C_i)$$

"Global Fit" = combination of $b \to s + (\gamma,\, \ell^+\ell^-)$ observables

Parameters of interest

$$\vec{\theta} = (C_i)$$

Nuisance parameters

$\vec{\nu}$

1) process-specific

   FF's, decay const's,
   LCDA pmr's,
   sub-leading Λ/m_b,
   renorm. scales: $\mu_{b,0}$

2) general

   quark masses, CKM, ...

“Global Fit” = combination of $b \to s + (\gamma, \ell^+\ell^-)$ observables

Parameters of interest

$$\vec{\theta} = (C_i)$$

Nuisance parameters

$\vec{\nu}$

1) process-specific

FF’s, decay const’s,
LCDA pmr’s,
sub-leading Λ/m_b,
renorm. scales: $\mu_{b,0}$

2) general

quark masses, CKM, . . .

Observables

1) observables

$$O(\vec{\theta}, \vec{\nu})$$

depend usually on sub-set of $\vec{\theta}$ and $\vec{\nu}$

2) experimental data for each observable

$$\text{pdf}(O = o)$$

$\Rightarrow$ probability distribution of values o

"Global Fit" = combination of $b \to s + (\gamma,\ \ell^+\ell^-)$ observables

Parameters of interest

$$\vec{\theta} = (C_i)$$

Nuisance parameters

$\vec{\nu}$

1) process-specific

FF's, decay const's,
LCDA pmr's,
sub-leading Λ/m_b,
renorm. scales: $\mu_{b,0}$

2) general

quark masses, CKM, ...

Observables

1) observables

$$O(\vec{\theta}, \vec{\nu})$$

depend usually on sub-set of $\vec{\theta}$ and $\vec{\nu}$

2) experimental data for each observable

$$\text{pdf}(O = o)$$

$\Rightarrow$ probability distribution of values o

Fit strategies: 1) Put theory uncertainties in likelihood:

$$\chi^2 = \sum \frac{(O_{\text{ex}} - O_{\text{th}})^2}{\sigma_{\text{ex}}^2 + \sigma_{\text{th}}^2}$$

- sample $\vec{\theta}$-space (grid, Markov Chain, importance sampling...)
- theory uncertainties of O_i at each $(\vec{\theta})_i$: vary $\vec{\nu}$ within some ranges $\Rightarrow \sigma_{\text{th}}(O[(\vec{\theta})_i])$
- use Frequentist or Bayesian method $\Rightarrow$ 68 & 95 % (CL or probability) regions of $\vec{\theta}$

“Global Fit” = combination of $b \to s + (\gamma, \ell^+\ell^-)$ observables

Parameters of interest

$$\vec{\theta} = (C_i)$$

Nuisance parameters

$\vec{\nu}$

1) process-specific

FF’s, decay const’s,
LCDA pmr’s,
sub-leading Λ/m_b,
renorm. scales: $\mu_{b,0}$

2) general

quark masses, CKM, ...

Observables

1) observables

$$O(\vec{\theta}, \vec{\nu})$$

depend usually on sub-set of $\vec{\theta}$ and $\vec{\nu}$

2) experimental data for each observable

$$\text{pdf}(O = o)$$

$\Rightarrow$ probability distribution of values o

Fit strategies: 2) Fit also nuisance parameters:

- sample $(\vec{\theta} \times \vec{\nu})$-space (grid, Markov Chain, importance sampling...)
- accounts for theory uncertainties by fitting also $(\vec{\nu})_i$
- use Frequentist or Bayesian method $\Rightarrow$ 68 & 95 % (CL or probability) regions of $\vec{\theta}$ and $\vec{\nu}$

Recent “Global Fit’s” after EPS-HEP 2013 Conference

DGMV	=	Descotes-Genon/Matias/Virto	[arXiv:1307.5683 + 1311.3876]	χ^2-frequentist
AS	=	Altmannshofer/Straub	[arXiv:1308.1501]	χ^2-fit
BBvD	=	Beaujean/Bobeth/van Dyk	[arXiv:1310.2478]	Bayesian
HLMW	=	Horgan/Liu/Meinel/Wingate	[arXiv:1310.3887]	χ^2-fit

Recent "Global Fit's" after EPS-HEP 2013 Conference

DGMV	=	Descotes-Genon/Matias/Virto	[arXiv:1307.5683 + 1311.3876]	χ^2-frequentist
AS	=	Altmannshofer/Straub	[arXiv:1308.1501]	χ^2-fit
BBvD	=	Beaujean/Bobeth/van Dyk	[arXiv:1310.2478]	Bayesian
HLMW	=	Horgan/Liu/Meinel/Wingate	[arXiv:1310.3887]	χ^2-fit

Theory predictions

@ low q^2: $B \to K^* \ell\bar{\ell}$, $B \to K\ell\bar{\ell}$, $B \to K^*\gamma$

DGMV, AS, BBvD: based on QCDF [Beneke/Feldmann/Seidel hep-ph/0106067 + 0412400]
(HLMW only uses high-q^2 data)

@ high q^2: $B \to K^* \ell\bar{\ell}$, $B \to K\ell\bar{\ell}$

DGMV, AS, BBvD, HLMW: based on local OPE
[Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

DGMV, AS, BBvD: LCSR $B \to K^*$ FF-results extrapolated from low q^2

HLMW: use their own lattice $B \to K^*$ FF predictions [arXiv:1310.3722]

Recent "Global Fit's" after EPS-HEP 2013 Conference

DGMV	=	Descotes-Genon/Matias/Virto	[arXiv:1307.5683 + 1311.3876]	χ^2-frequentist
AS	=	Altmannshofer/Straub	[arXiv:1308.1501]	χ^2-fit
BBvD	=	Beaujean/Bobeth/van Dyk	[arXiv:1310.2478]	Bayesian
HLMW	=	Horgan/Liu/Meinel/Wingate	[arXiv:1310.3887]	χ^2-fit

Theory predictions

@ low q^2: $B \to K^* \ell\bar{\ell}$, $B \to K\ell\bar{\ell}$, $B \to K^*\gamma$

DGMV, AS, BBvD: based on QCDF [Beneke/Feldmann/Seidel hep-ph/0106067 + 0412400]
(HLMW only uses high-q^2 data)

@ high q^2: $B \to K^* \ell\bar{\ell}$, $B \to K\ell\bar{\ell}$

DGMV, AS, BBvD, HLMW: based on local OPE
[Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

DGMV, AS, BBvD: LCSR $B \to K^*$ FF-results extrapolated from low q^2

HLMW: use their own lattice $B \to K^*$ FF predictions [arXiv:1310.3722]

Theory uncertainties

DGMV, AS, HLMW: combining theoretical and experimental uncertainties
$\Rightarrow$ included in likelihood

BBvD: most relevant parameters included in the fit as nuisance parameters

Which data is used?

q^2 Binning

	q^2-Bins [GeV^2]
lo	[1, 6]
LO	[0, 2]
	[2, 4.3]
	[4.3, 8.68]
hi	[14.18, 16]
	[16, 19]

DGMV: only LHCb data for $B \to K^* \ell\bar{\ell}$

AS, BBvD, HLMW:
use all available data from Belle, Babar, CDF, LHCb, CMS, ATLAS

decay	obs	DGMV	AS	BBvD	HLMW
$B \to X_s\gamma$	Br	✓	✓	✓	
	A_{CP}		✓		
$B \to K^*\gamma$	Br			✓	
	$S(C)$	✓	✓	✓(✓)	
	A_I	✓			
$B_s \to \mu\bar{\mu}$	Br	✓	✓	✓	
$B \to X_s\ell\bar{\ell}$	Br	lo	lo+hi	lo	
$B \to K\ell\bar{\ell}$	Br		lo+hi	lo+hi	
$B \to K^*\ell\bar{\ell}$	Br		lo+hi	lo+hi	hi
	F_L		lo+hi	lo+hi	hi
	$A_{\rm FB}$	LO+hi	lo+hi	lo+hi†	hi
	$P_{1,2}$, $P'_{4,5,6}$	LO+hi		lo+hi†	
	P'_8	LO+hi			
	$S_{3,4,5}$		lo+hi		hi
	A_9		lo+hi		
$B_s \to \phi\ell\bar{\ell}$	Br, F_L, S_3				hi

† if P_2 is available then $A_{\rm FB}$ is not used: LHCb

"Only $B \to K^* \ell^+ \ell^-$": DGMV

with 3 q^2-bins @ low q^2

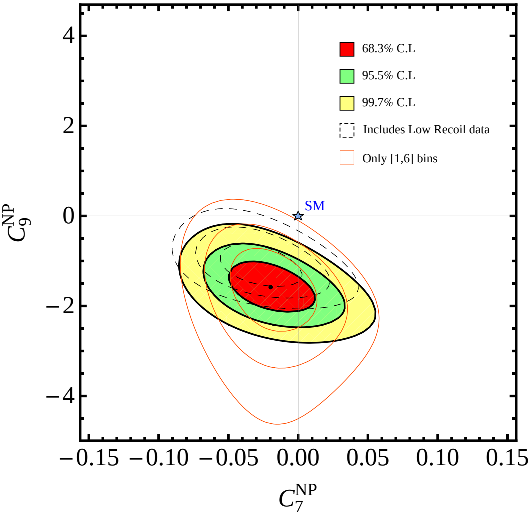
1) only low q^2:
   A_{FB}, P_2 and P_5' prefer:

   $C_9^{NP} \approx -1.6$

2) adding high q^2:
   due to $q^2 \in [14.18, 16.0]$ GeV^2 bin

   $C_9^{NP} \approx -1.2$

3) only $C_7^{NP} \neq 0$ beneficial,
   NO real need for $C_{7',9',10'}$,
   however $C_{9'} < 0$
   preferred



“Adding $B \to K\ell^+\ell^-$”: AS

$\Rightarrow$ 3 main tensions between data and SM:

A) F_L @ low q^2 (from Babar and ATLAS)

B) P_5'/S_5 @ low q^2

C) P_4'/S_4 @ high q^2
($\Leftarrow$ even not resolvable with $C_{7',9',10'} \neq 0$)

1) $C_{7,9}^{NP} \neq 0$ can reduce tension for F_L and S_5,
but not as good as:

2) C_9^{NP} with $C_{9'}$ (or $C_{10'}$)
$B \to K\ell\bar{\ell}$ requires $C_{9'} > 0$ (or $C_{10'} < 0$)

3) Fit does not improve much when allowing all
$C_{i(')} \neq 0 \to$ best fit:

$$C_7^{NP} = -0.03, \quad C_9^{NP} = -0.9, \quad C_{10}^{NP} = -0.1,$$
$$C_{7'} = -0.11, \quad C_{9'} = +0.7, \quad C_{10'} = -0.2$$

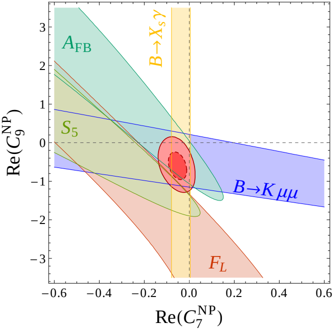
“Adding $B \to K\ell^+\ell^-$”: AS

$\Rightarrow$ 3 main tensions between data and SM:

A) F_L @ low q^2 (from Babar and ATLAS)

B) P_5'/S_5 @ low q^2

C) P_4'/S_4 @ high q^2
($\Leftarrow$ even not resolvable with $C_{7',9',10'} \neq 0$)



1) $C_{7,9}^{NP} \neq 0$ can reduce tension for F_L and S_5, but not as good as:

2) C_9^{NP} with $C_{9'}$ (or $C_{10'}$)
$B \to K\ell\bar{\ell}$ requires $C_{9'} > 0$ (or $C_{10'} < 0$)

3) Fit does not improve much when allowing all $C_{i(')} \neq 0 \to$ best fit:

$$C_7^{NP} = -0.03, \quad C_9^{NP} = -0.9, \quad C_{10}^{NP} = -0.1,$$
$$C_{7'} = -0.11, \quad C_{9'} = +0.7, \quad C_{10'} = -0.2$$

“Adding $B \to K\ell^+\ell^-$”: AS

$\Rightarrow$ 3 main tensions between data and SM:

A) F_L @ low q^2 (from Babar and ATLAS)

B) P_5'/S_5 @ low q^2

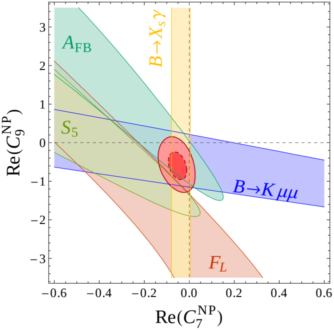
C) P_4'/S_4 @ high q^2
($\Leftarrow$ even not resolvable with $C_{7',9',10'} \neq 0$)

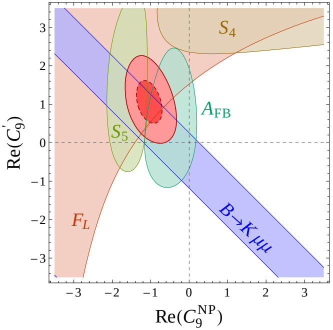
1) $C_{7,9}^{NP} \neq 0$ can reduce tension for F_L and S_5, but not as good as:

2) C_9^{NP} with $C_{9'}$ (or $C_{10'}$)
$B \to K\ell\bar{\ell}$ requires $C_{9'} > 0$ (or $C_{10'} < 0$)

3) Fit does not improve much when allowing all $C_{i(')} \neq 0 \to$ best fit:

$$C_7^{NP} = -0.03, \quad C_9^{NP} = -0.9, \quad C_{10}^{NP} = -0.1,$$
$$C_{7'} = -0.11, \quad C_{9'} = +0.7, \quad C_{10'} = -0.2$$





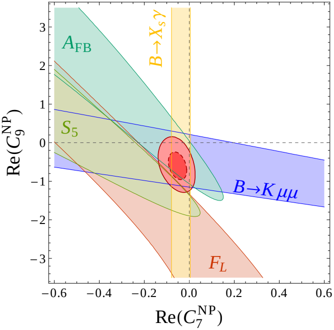
“Adding $B \to K\ell^+\ell^-$”: AS

$\Rightarrow$ 3 main tensions between data and SM:

A) F_L @ low q^2 (from Babar and ATLAS)

B) P_5'/S_5 @ low q^2

C) P_4'/S_4 @ high q^2
($\Leftarrow$ even not resolvable with $C_{7',9',10'} \neq 0$)

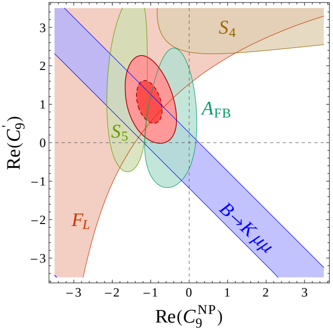


1) $C_{7,9}^{NP} \neq 0$ can reduce tension for F_L and S_5,
but not as good as:

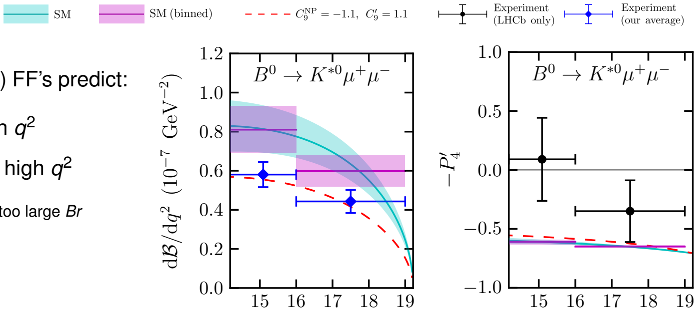
2) C_9^{NP} with $C_{9'}$ (or $C_{10'}$)
$B \to K\ell\bar\ell$ requires $C_{9'} > 0$ (or $C_{10'} < 0$)

3) Fit does not improve much when allowing all $C_{i(')} \neq 0 \to$ best fit:

$$C_7^{NP} = -0.03, \quad C_9^{NP} = -0.9, \quad C_{10}^{NP} = -0.1,$$
$$C_{7'} = -0.11, \quad C_{9'} = +0.7, \quad C_{10'} = -0.2$$



"Only $B \to K^* \ell^+ \ell^-$ @ high q^2" with $B \to K^*$ lattice FF's: HLMW



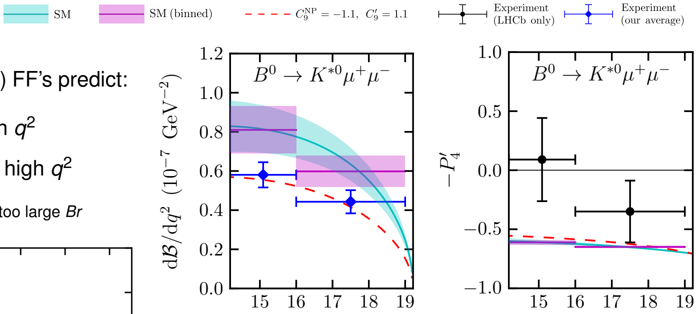
$\Rightarrow$ $B \to K^*$ (and $B_s \to \phi$) FF's predict:

A) too large Br @ high q^2

B) too small P_4'/S_4 @ high q^2

also ($B_s \to \phi$) FF's predict too large Br

"Only $B \to K^* \ell^+ \ell^-$ @ high q^2" with $B \to K^*$ lattice FF's: HLMW

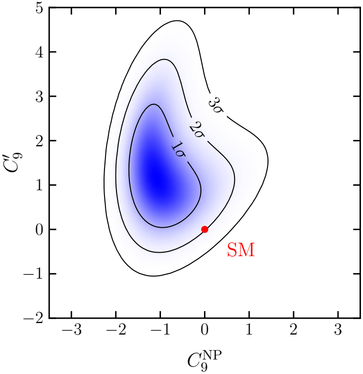


$\Rightarrow$ $B \to K^*$ (and $B_s \to \phi$) FF's predict:

A) too large Br @ high q^2

B) too small P_4'/S_4 @ high q^2

also ($B_s \to \phi$) FF's predict too large Br



1) only high q^2 data of $B \to K^* \ell\bar{\ell}$ & $B_s \to \phi \ell\bar{\ell}$

2) consider only $C_9^{NP} - C_9'$ scenario

3) best fit point:

$$C_9^{NP} = -1.1 \pm 0.5, \qquad C_{9'} = +1.1 \pm 0.9$$

and only highest $q^2 \in [16, 19]$ GeV^2 bin:

$$C_9^{NP} = -1.1 \pm 0.7, \qquad C_{9'} = +0.4 \pm 0.7$$

"Fitting also all the nuisance": BBvD

Fit nuisance parameters . . .

A) describing q^2-dependence of form factors

- $B \to K$: $2\times$ $\to$ prior from LCSR + Lattice
- $B \to K^*$: $6\times$ $\to$ prior from LCSR (NO Lattice)

B) of naive parametrisation of subleading corrections

- $B \to K$: $2\times$ @ low and high q^2
- $B \to K^*$: $6\times$ @ low q^2 and $3\times$ @ high q^2

priors: about 15%$\sim \Lambda_{\rm QCD}/m_b$ of leading amplitude

C) CKM, quark masses, . . .

. . . in total 28 nuisance parameters

“Fitting also all the nuisance”: BBvD

Fit nuisance parameters . . .

A) describing q^2-dependence of form factors
- $B \to K$: $2\times$ $\to$ prior from LCSR + Lattice
- $B \to K^*$: $6\times$ $\to$ prior from LCSR (NO Lattice)

B) of naive parametrisation of subleading corrections
- $B \to K$: $2\times$ @ low and high q^2
- $B \to K^*$: $6\times$ @ low q^2 and $3\times$ @ high q^2

priors: about 15%$\sim \Lambda_{\rm QCD}/m_b$ of leading amplitude

C) CKM, quark masses, . . .

. . . in total 28 nuisance parameters

Fits in the SM

1) SM = only nuisance parameters

and model-independent scenarios

2) $\text{SM}_{7,9,10}$ = $C^{NP}_{7,9,10} \neq 0$

3) SM+SM' = $C^{NP}_{7,9,10} \neq 0$ and $C_{7',9',10'} \neq 0$

4) $\text{SM+SM'}_{9,9'}$ = $C^{NP}_{9} \neq 0$ and $C_{9'} \neq 0$

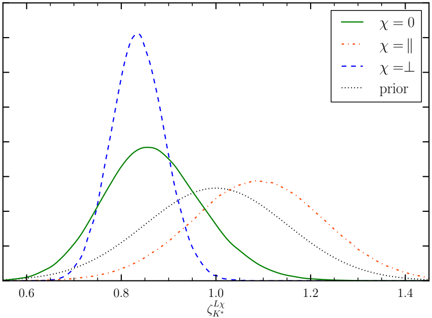
“Fitting also all the nuisance”: BBvD

SM = fit of nuisance parameters

$\Rightarrow$ some subleading $B \to K^*$ corrections around

- $\sim -(15 - 20)\%$ for $\chi = \perp, 0$
- $\sim +10\%$ for $\chi = \parallel$

@ low q^2 level of amplitude
with gaussian priors of $1\sigma \sim \Lambda_{\rm QCD}/m_b \sim 15\%$



“Fitting also all the nuisance”: BBvD

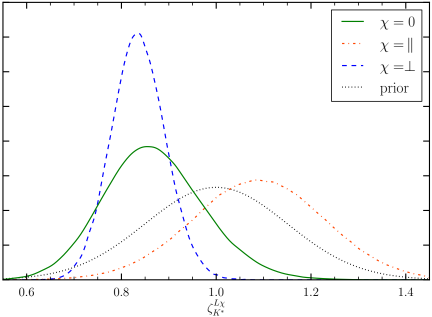
SM = fit of nuisance parameters

$\Rightarrow$ some subleading $B \to K^*$ corrections around

$\sim -(15-20)\%$ for $\chi = \perp, 0$

$\sim +10\%$ for $\chi = \|$

@ low q^2 level of amplitude
with gaussian priors of $1\sigma \sim \Lambda_{\rm QCD}/m_b \sim 15\%$



$\Rightarrow$ 6 experimental measurements with pull values $> 2\sigma$ (out of 92) @ best fit point:

Belle	:	$\langle Br \rangle_{[16,19]}$ $(+2.6\sigma)$	
Babar	:	$\langle F_L \rangle_{[1,6]}$ (-3.5σ)	
LHCb	:	$\langle P'_4 \rangle_{[14,16]}$ (-2.4σ)	$\langle P'_5 \rangle_{[1,6]}$ $(+2.1\sigma)$
ATLAS	:	$\langle A_{\rm FB} \rangle_{[16,19]}$ $(+2.2\sigma)$	$\langle F_L \rangle_{[1,6]}$ (-2.6σ)

not yet published

"Fitting also all the nuisance": BBvD

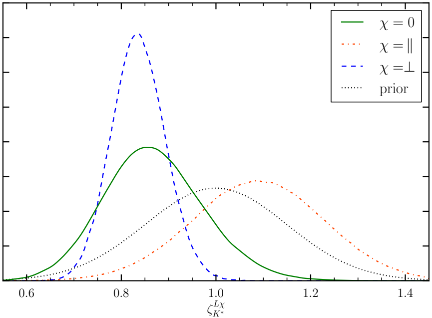
SM = fit of nuisance parameters

$\Rightarrow$ some subleading $B \to K^*$ corrections around

$\sim -(15-20)\%$ for $\chi = \perp, 0$

$\sim +10\%$ for $\chi = \parallel$

@ low q^2 level of amplitude
with gaussian priors of $1\sigma \sim \Lambda_{\rm QCD}/m_b \sim 15\%$



$\Rightarrow$ 6 experimental measurements with pull values $> 2\sigma$ (out of 92) @ best fit point:

Belle	:	$\langle Br \rangle_{[16,19]}$ $(+2.6\sigma)$	
Babar	:	$\langle F_L \rangle_{[1,6]}$ (-3.5σ)	
LHCb	:	$\langle P_4' \rangle_{[14,16]}$ (-2.4σ)	$\langle P_5' \rangle_{[1,6]}$ $(+2.1\sigma)$
ATLAS	:	$\langle A_{\rm FB} \rangle_{[16,19]}$ $(+2.2\sigma)$	$\langle F_L \rangle_{[1,6]}$ (-2.6σ)

not yet published

$\Rightarrow$ form factors

data prefers higher FF's in SM+SM' than $SM_{7,9,10}$

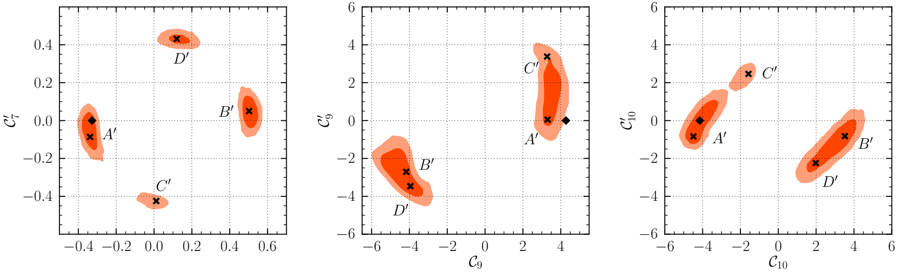
$\rightarrow$ consistent with lattice

	prior	SM	$SM_{7,9,10}$	SM+SM'
$V(0)$	$0.36^{+0.23}_{-0.12}$	$0.38^{+0.04}_{-0.02}$	$0.38^{+0.03}_{-0.03}$	$0.38^{+0.04}_{-0.03}$
$A_1(0)$	$0.25^{+0.16}_{-0.10}$	$0.24^{+0.03}_{-0.02}$	$0.24^{+0.03}_{-0.03}$	$0.28^{+0.04}_{-0.03}$
$A_2(0)$	$0.23^{+0.19}_{-0.10}$	$0.23^{+0.04}_{-0.04}$	$0.22^{+0.05}_{-0.04}$	$0.27^{+0.06}_{-0.05}$

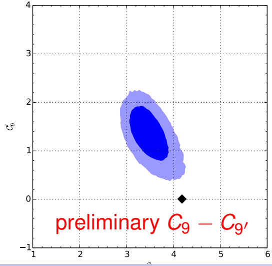
“Fitting also all the nuisance”: BBvD

SM+SM’

4 solutions with posterior masses: $A' = 39\%$, $B' = 41\%$, $C' = 5\%$, $D' = 15\%$



SM+SM’$_{9,9'}$



SM = (◆), best fit point = (×)

“Fitting also all the nuisance”: BBvD

Model comparison

of models M_1 and M_2 with

$$\frac{P(M_1|D)}{P(M_2|D)} = B(D|M_1, M_2)\frac{P(M_1)}{P(M_2)}$$

- $P(M_i)$ = priors of the models M_i
- Bayes factor:

$$B(D|M_1, M_2) \equiv \frac{P(D|M_1)}{P(D|M_2)}$$

!!! Models with more parameters are disfavored by larger prior volume,
unless they improve the fit substantially

“Fitting also all the nuisance”: BBvD

Model comparison

of models M_1 and M_2 with

$$\frac{P(M_1|D)}{P(M_2|D)} = B(D|M_1, M_2)\frac{P(M_1)}{P(M_2)}$$

- $P(M_i)$ = priors of the models M_i
- Bayes factor:

$$B(D|M_1, M_2) \equiv \frac{P(D|M_1)}{P(D|M_2)}$$

!!! Models with more parameters are disfavored by larger prior volume,
unless they improve the fit substantially

Assuming only SM-like signs of Wilson coefficients
= reducing prior volume of Wilson coefficients to include only $A^{(\prime)}$

$\Rightarrow$ corresponds to flat priors:

	$SM_{7,9,10} : SM$	SM+SM' : SM	$SM+SM'_{9,9'} : SM$
$B(D|M_1, M_2)^\dagger$	1 : 100	1 : 22	9 : 1

$$C_{7^{(\prime)}} \in [-0.5, 0.5]$$

$$C_{9^{(\prime)},10^{(\prime)}} \in [-3.8, 3.8]$$

† H. Jeffreys interpretation of $B(D|M_1, M_2)$ as strength of evidence in favour of M_2:
1:3 < barely worth mentioning, 1:10 < substantial, 1:30 < strong, 1:100 < very strong, > 1:100 decisive.

Summary

- 4 analyses → many differences:
  1) choice of data
  2) choice of theory uncertainties (subleading, high q^2, FF's)

  ⇒ still: consistent picture in fits

- $B \to K^* \ell\bar{\ell}$ low-q^2 data prefers $C_9^{NP} < 0$, not only from P_5'
- $B \to K^* \ell\bar{\ell}$ high-q^2 data with $B \to K^*$ FF's prefers $C_9^{NP} < 0$ & $C_{9'} > 0$

- in combination with $B \to K\ell\bar{\ell}$ can drive $C_{9',10'} \neq 0$

- subleading corrections @ low $q^2 \neq 0$, but within $\Lambda_{\rm QCD}/m_b$ expectation

- from model comparison: scenarios SM+SM' favoured over SM

EOS = Flavour tool @ TU Dortmund by Danny van Dyk et al.

Download @ http://project.het.physik.tu-dortmund.de/eos/

Issues ?!

Perhaps with data:

- fluctuations in the data

  $\Rightarrow$ new results will be available from

  1) Belle (final reprocessed)
  2) LHCb ($1 \to 3$ fb^{-1})
  3) CMS and ATLAS ($5 \to 25$ fb^{-1})
  4) Babar F_L, $A_{\rm FB}$ not yet published

  hopefully within next year

and/or the theory:

- theory @ high q^2

  1) local OPE is not reliable (large duality violation)

     $\Rightarrow$ some predictions of OPE can be tested experimentally

     [CB/Hiller/van Dyk arXiv:1006.5013 + 1212.2321]
  2) q^2-binning in exp. data not yet optimal for OPE?
  3) $B \to K^*$ form factors from lattice too high and/or underestimated systematics?

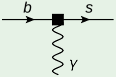
- theory @ low q^2

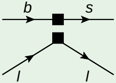
  1) for subleading corrections $\Lambda_{\rm QCD}/m_b$
  2) large long-distance $c\bar{c}$ contributions

Backup Slides

Exclusive $B \to K^*(\to K\pi)\,\ell^+\ell^-$

Hadronic amplitude $B \to K^*(\to K\pi)\,\ell^+\ell^-$ — neglecting 4-quark operators

$\mathcal{M} = \langle K\pi|\; C_7 \times$ 

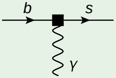
$+ \;\; C_{9,10} \times$ 

$|B\rangle$

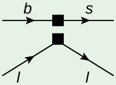
Exclusive $B \to K^*(\to K\pi)\,\ell^+\ell^-$

Hadronic amplitude $B \to K^*(\to K\pi)\,\ell^+\ell^-$ — neglecting 4-quark operators

$$\mathcal{M} = \langle K\pi|\; C_7 \times$$



$$+ \; C_{9,10} \times$$



$$|B\rangle$$

$\mathcal{M}$ may expressed in terms of transversity amplitudes of K^* ($m_\ell = 0$)

. . . using narrow width approximation & intermediate K^* on-shell

$\Rightarrow$ “just” requires $B \to K^*$ form factors V, $A_{1,2}$, $T_{1,2,3}$:

$$A_\perp^{L,R} \sim \sqrt{2\,\lambda}\left[(C_9 \mp C_{10})\frac{V}{M_B + M_{K*}} + \frac{2\,m_b}{q^2}C_7\,T_1\right],$$

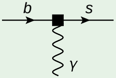
$$A_\parallel^{L,R} \sim -\sqrt{2}\,(M_B^2 - M_{K*}^2)\left[(C_9 \mp C_{10})\frac{A_1}{M_B - M_{K*}} + \frac{2\,m_b}{q^2}C_7\,T_2\right],$$

$$A_0^{L,R} \sim -\frac{1}{2\,M_{K*}\sqrt{q^2}}\left\{(C_9 \mp C_{10})\Big[\ldots A_1 + \ldots A_2\Big] + 2\,m_b C_7\Big[\ldots T_2 + \ldots T_3\Big]\right\}$$

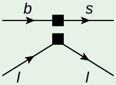
Exclusive $B \to K^*(\to K\pi)\,\ell^+\ell^-$

Hadronic amplitude $B \to K^*(\to K\pi)\,\ell^+\ell^-$ — including 4-quark operators

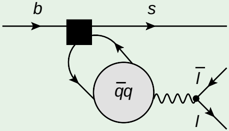
$\mathcal{M} = \langle K\pi|\; C_7 \times$



$+ \; C_{9,10} \times$



$+ \sum_i C_i \times$



$|B\rangle$

. . . but 4-Quark operators and $\mathcal{O}_{8g}$ have to be included

- current-current $b \to s + (\bar{u}u,\, \bar{c}c)$
- QCD-penguin operators $b \to s + \bar{q}q$ $(q = u, d, s, c, b)$

$\Rightarrow$ large peaking background around certain $q^2 = (M_{J/\psi})^2,\ (M_{\psi'})^2$:

$B \to K^{(*)}(\bar{q}q) \to K^{(*)}\bar{\ell}\ell$

q^2 - regions in $b \to s\,\ell^+\ell^-$

$K^{(*)}$-energy in B-rest frame: $E_{K^{(*)}} = (M_B^2 + M_{K^{(*)}}^2 - q^2)/(2\,M_B)$

$\Rightarrow$ Two regions in q^2 where theory can give reliable predictions beyond naive factorization

q^2-region	low-q^2: $q^2 \ll M_B^2$	high-q^2: $q^2 \sim M_B^2$
$K^{(*)}$-recoil	large recoil: $E_{K^{(*)}} \sim M_B/2$	low recoil: $E_{K^{(*)}} \sim M_{K^{(*)}} + \Lambda_{\rm QCD}$
theory method	QCDF, nl OPE: $q^2 \in [1, 6]$ GeV^2	OPE + HQET: $q^2 \gtrsim (14\ldots15)$ GeV^2

[QCDF: Beneke/Feldmann/Seidel hep-ph/0106067 + 0412400]

[non-local OPE: Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945 + 1211.0234]

[local OPE: Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

$\Rightarrow$ $\bar{c}c$ vetoed in experiment

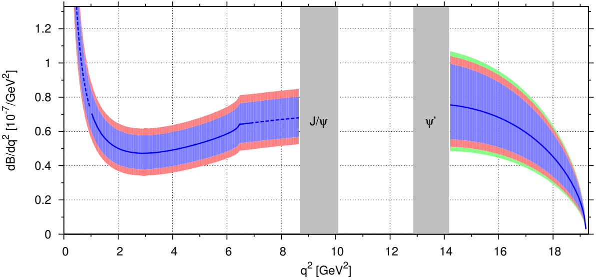
$d Br[B \to K^* \ell^+\ell^-]/dq^2$

$\Rightarrow$ light resonances $q^2 \lesssim 1$ GeV^2 not vetoed

small for CP-aver. obs's
relevant for CP-asy's

[Jäger/Martin-Camalich 1212.2263]

[Khodjamirian/Mannel/Wang 1211.0234]



Low-q^2 = Large Recoil

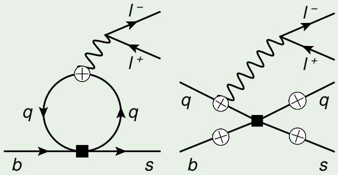
QCD Factorisation (QCDF)

[Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]

= (large recoil + heavy quark) limit [also Soft Collinear ET (SCET)]

$$\left\langle \bar{\ell}\ell\, K_a^* \,\middle|\, H_{\rm eff}^{(i)} \,\middle|\, B \right\rangle \sim$$

$$C_a^{(i)} \times \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K*} + \mathcal{O}(\Lambda_{\rm QCD}/m_b)$$

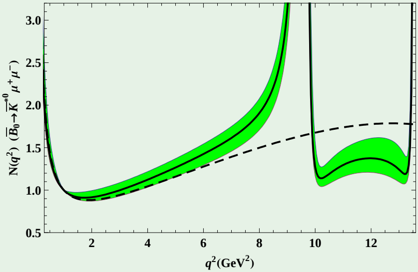


$C_a^{(i)}$, $T_a^{(i)}$: perturbative kernels in α_s ($a = \perp, \parallel$, $i = u, t$)

ϕ_B, $\phi_{a,K*}$: B– and K_a^*–distribution amplitudes

$c\bar{c}$-contributions

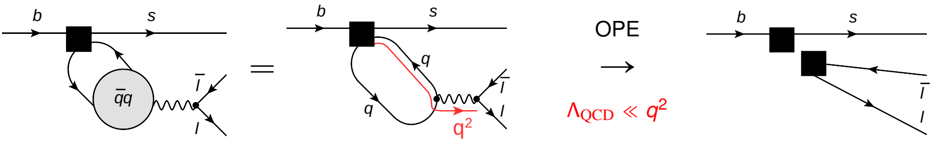
[Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]



- OPE near light-cone incl. soft-gluon emission (non-local operator) for $q^2 \lesssim 4$ GeV^2 $\ll 4 m_c^2$
- hadronic dispersion relation using measured $B \to K^{(*)}(\bar{c}c)$ amplitudes at $q^2 \gtrsim 4$ GeV^2
- $B \to K^{(*)}$ form factors from LCSR
- up to (15-20) % in rate for $1 < q^2 < 6$ GeV^2

High-q^2 = Low Recoil

Hard momentum transfer $(q^2 \sim M_B^2)$ through $(\bar{q}q) \to \bar{\ell}\ell$ allows local OPE



$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{L}^{\rm eff}(0), j_\mu^{\rm em}(x)\} | \bar{B}\rangle [\bar{\ell}\gamma^\mu \ell]$$

$$= \left(\sum_a \mathcal{C}_{3a} \mathcal{Q}_{3a}^\mu + \sum_b \mathcal{C}_{5b} \mathcal{Q}_{5b}^\mu + \sum_c \mathcal{C}_{6c} \mathcal{Q}_{6c}^\mu + \mathcal{O}(dim > 6)\right) [\bar{\ell}\gamma_\mu \ell]$$

Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118

Leading $dim = 3$ operators: $\langle \bar{K}^* | \mathcal{Q}_{3,a} | \bar{B}\rangle \sim$ usual $B \to K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$

$$\mathcal{Q}_{3,1}^\mu = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) [\bar{s}\,\gamma_\nu (1-\gamma_5)\, b] \qquad \to \qquad C_9 \to C_9^{\rm eff}, \qquad (V, A_{1,2})$$

$$\mathcal{Q}_{3,2}^\mu = \frac{i m_b}{q^2} q_\nu\, [\bar{s}\,\sigma_{\nu\mu} (1+\gamma_5)\, b] \qquad \to \qquad C_7 \to C_7^{\rm eff}, \qquad (T_{1,2,3})$$

High-q^2 = Low Recoil

[Beylich/Buchalla/Feldmann arXiv:1101.5118]

$dim = 3$ α_s matching corrections are also known

$m_s \neq 0$ 2 additional $dim = 3$ operators, suppressed with $\alpha_s m_s/m_b \sim 0.5$ %,
NO new form factors

$dim = 4$ absent

$dim = 5$ suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2$ %,
explicite estimate @ $q^2 = 15$ GeV^2: $< 1\%$

$dim = 6$ suppressed by $(\Lambda_{\rm QCD}/m_b)^3 \sim 0.2$ % and small QCD-penguin's: $C_{3,4,5,6}$
spectator quark effects: from weak annihilation

beyond OPE duality violating effects

- based on Shifman model for c-quark correlator + fit to recent BES data
- ± 2 % for integrated rate $q^2 > 15$ GeV^2

$\Rightarrow$ OPE of exclusive $B \to K^{(*)} \ell^+ \ell^-$ predicts small sub-leading contributions !!!

BUT, still missing $B \to K^{(*)}$ form factors @ high-q^2
for predictions of angular observables J_i

High-q^2: OPE + HQET

Framework developed by Grinstein/Pirjol hep-ph/0404250

1) OPE in $\Lambda_{\rm QCD}/Q$ with $Q = \{m_b, \sqrt{q^2}\}$ + matching on HQET + expansion in m_c

$$\mathcal{M}[\bar B \to \bar K^* + \bar\ell\ell] \sim \frac{8\pi}{q^2} \sum_{i=1}^{6} \mathcal{C}_i(\mu)\, \mathcal{T}_\alpha^{(i)}(q^2,\mu)\, [\bar\ell\gamma^\alpha\ell]$$

$$\mathcal{T}_\alpha^{(i)}(q^2,\mu) = i\int d^4x\, e^{iq\cdot x} \langle \bar K^* | T\{\mathcal{O}_i(0), j_\alpha^{\rm em}(x)\}|\bar B\rangle$$

$$= \sum_{k\geqslant -2} \sum_j C_{i,j}^{(k)} \langle \mathcal{Q}_{j,\alpha}^{(k)} \rangle$$

$\mathcal{Q}_{j,\alpha}^{(k)}$	power	$\mathcal{O}(\alpha_s)$
$\mathcal{Q}_{1,2}^{(-2)}$	1	$\alpha_s^0(Q)$
$\mathcal{Q}_{1-5}^{(-1)}$	$\Lambda_{\rm QCD}/Q$	$\alpha_s^1(Q)$
$\mathcal{Q}_{1,2}^{(0)}$	m_c^2/Q^2	$\alpha_s^0(Q)$
$\mathcal{Q}_{j>3}^{(0)}$	${\Lambda_{\rm QCD}}^2/Q^2$	$\alpha_s^0(Q)$
$\mathcal{Q}_{i}^{(2)}$	m_c^4/Q^4	$\alpha_s^0(Q)$

included,
unc. estimate by naive pwr cont.

2) HQET FF-relations at sub-leading order + α_s corrections in leading order

$$T_1(q^2) = \kappa\, V(q^2), \qquad T_2(q^2) = \kappa\, A_1(q^2), \qquad T_3(q^2) = \kappa\, A_2(q^2)\frac{M_B^2}{q^2},$$

$$\kappa = \left(1 + \frac{2D_0^{(v)}(\mu)}{C_0^{(v)}(\mu)}\right)\frac{m_b(\mu)}{M_B}$$

can express everything in terms of QCD FF's $V, A_{1,2}$ @ $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/Q)$!!!

Angular observables

$$J_i(q^2) \sim \{\text{Re, Im}\} \left[A_m^{L,R} \left(A_n^{L,R} \right)^* \right]$$

$$\sim \sum_a (C_a F_a) \sum_b (C_b F_b)^*$$

$A_m^{L,R} \ldots K^*$-transversity amplitudes $m = \perp, \parallel, 0$

$C_a \ldots$ short-distance coefficients

$F_a \ldots$ form factors

Angular observables

$$J_i(q^2) \sim \{\mathrm{Re}, \mathrm{Im}\} \left[A_m^{L,R} \left(A_n^{L,R} \right)^* \right]$$

$$\sim \sum_a (C_a F_a) \sum_b (C_b F_b)^*$$

$A_m^{L,R} \ldots K^*$-transversity amplitudes $m = \perp, \|, 0$

$C_a \ldots$ short-distance coefficients

$F_a \ldots$ form factors

simplify when using form factor relations:

low K^* recoil limit: $E_{K^*} \sim M_{K^*} \sim \Lambda_{\mathrm{QCD}}$ [Isgur/Wise PLB232 (1989) 113, PLB237 (1990) 527]

$$T_1 \approx V, \qquad T_2 \approx A_1, \qquad T_3 \approx A_2 \frac{M_B^2}{q^2}$$

large K^* recoil limit: $E_{K^*} \sim M_B$ [Charles et al. hep-ph/9812358, Beneke/Feldmann hep-ph/0008255]

$$\xi_\perp \equiv \frac{M_B}{M_B + M_{K^*}} V \approx \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 \approx T_1 \approx \frac{M_B}{2E_{K^*}} T_2$$

$$\xi_\| \equiv \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 - \frac{M_B - M_{K^*}}{M_{K^*}} A_2 \approx \frac{M_B}{2E_{K^*}} T_2 - T_3$$

Sub-leading corrections to TransAmp's

$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil

$$A_i^{L,R} \sim C^{L,R} \times f_i \qquad\qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$)

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K*}} V, \qquad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K*})\,A_1, \qquad f_0 = \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Sub-leading corrections to TransAmp's

$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil

FF symmetry breaking

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s) \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$) $\qquad C_7^{\rm SM} \approx -0.3,\ C_9^{\rm SM} \approx 4.2,\ C_{10}^{\rm SM} \approx -4.2$

$$f_\perp = \frac{\sqrt{2\hat\lambda}}{1+\hat M_{K*}} V, \qquad f_\parallel = \sqrt{2}\,(1+\hat M_{K*})\,A_1, \qquad f_0 = \frac{(1-\hat s - \hat M_{K*}^2)(1+\hat M_{K*})^2 A_1 - \hat\lambda\, A_2}{2\,\hat M_{K*}(1+\hat M_{K*})\sqrt{\hat s}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Sub-leading corrections to TransAmp's

$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil

FF symmetry breaking   OPE

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s) + \mathcal{O}\left(\lambda^2\right), \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$) $C_7^{\rm SM} \approx -0.3,\ C_9^{\rm SM} \approx 4.2,\ C_{10}^{\rm SM} \approx -4.2$

$$f_\perp = \frac{\sqrt{2\hat\lambda}}{1+\hat M_{K*}} V, \quad f_\parallel = \sqrt{2}\,(1+\hat M_{K*})\,A_1, \quad f_0 = \frac{(1-\hat s - \hat M_{K*}^2)(1+\hat M_{K*})^2 A_1 - \hat\lambda\, A_2}{2\,\hat M_{K*}(1+\hat M_{K*})\sqrt{\hat s}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Sub-leading corrections to TransAmp's

$\lambda = \Lambda_{\text{QCD}}/m_b \sim 0.15$

Low hadronic recoil

$\Rightarrow$ small, apart from possible duality violations

FF symmetry breaking OPE

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s) + \mathcal{O}\left(\lambda^2\right), \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2 m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$)

$C_7^{\text{SM}} \approx -0.3,\ C_9^{\text{SM}} \approx 4.2,\ C_{10}^{\text{SM}} \approx -4.2$

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K*}} V, \quad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K*})\,A_1, \quad f_0 = \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 A_1 - \hat{\lambda}\, A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Large hadronic recoil

$$A_{\perp,\parallel}^{L,R} \sim \pm C_\perp^{L,R} \times \xi_\perp + \mathcal{O}(\alpha_s, \lambda), \qquad A_0^{L,R} \sim C_\parallel^{L,R} \times \xi_\parallel + \mathcal{O}(\alpha_s, \lambda)$$

2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

$$C_\perp^{L,R} = (C_9 \mp C_{10}) + \frac{2 m_b M_B}{q^2} C_7, \qquad C_\parallel^{L,R} = (C_9 \mp C_{10}) + \frac{2\, m_b}{M_B} C_7,$$

Sub-leading corrections to TransAmp's

$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil $\Rightarrow$ small, apart from possible duality violations

FF symmetry breaking OPE

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s) + \mathcal{O}\left(\lambda^2\right), \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2 m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$)

$C_7^{\rm SM} \approx -0.3,\ C_9^{\rm SM} \approx 4.2,\ C_{10}^{\rm SM} \approx -4.2$

$$f_\perp = \frac{\sqrt{2\hat\lambda}}{1 + \hat M_{K*}} V, \qquad f_\parallel = \sqrt{2}\,(1 + \hat M_{K*})\, A_1, \qquad f_0 = \frac{(1 - \hat s - \hat M_{K*}^2)(1 + \hat M_{K*})^2 A_1 - \hat\lambda\, A_2}{2\, \hat M_{K*} (1 + \hat M_{K*}) \sqrt{\hat s}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Large hadronic recoil $\Rightarrow$ limited, end-point-divergences at $\mathcal{O}(\lambda)$

$$A_{\perp,\parallel}^{L,R} \sim \pm C_\perp^{L,R} \times \xi_\perp + \mathcal{O}(\alpha_s, \lambda), \qquad A_0^{L,R} \sim C_\parallel^{L,R} \times \xi_\parallel + \mathcal{O}(\alpha_s, \lambda)$$

2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

$$C_\perp^{L,R} = (C_9 \mp C_{10}) + \frac{2 m_b M_B}{q^2} C_7, \qquad C_\parallel^{L,R} = (C_9 \mp C_{10}) + \frac{2\, m_b}{M_B} C_7,$$