Bayesian fit of rare B decays with EOS

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New Physics at Belle II
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Outline

Physics case: Rare *B* decays

- Flavour-changing decays in the standard model (SM)
- Experimental results
- Effective Theory (EFT) of $|\Delta B| = |\Delta S| = 1$ decays
- From EFT towards observables

EOS: Rare *B* decays

- Fit strategy and general work flow
- Steering fits
- Implemented observables

EOS: Model-independent Fits

Physics case:

Rare B decays

Flavour changes in the Standard Model (SM)

$U_i = \{u, c, t\}$:

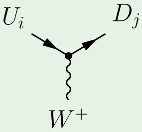
$Q_U = +2/3$

$D_j = \{d, s, b\}$:

$Q_D = -1/3$

$$\mathcal{L}_{\rm CC} = \frac{g_2}{\sqrt{2}} \, (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma^\mu P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_\mu$$

~ Cabibbo-Kobayashi-Maskawa (CKM) matrix



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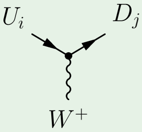

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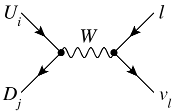
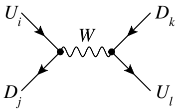
$\Rightarrow$ charged current: $Q_i \neq Q_j$





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$M_1 \to M_2 + \ell \nu_\ell$

$M_1 \to M_2 M_3$

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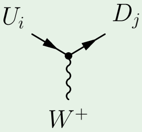

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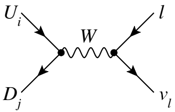

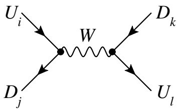

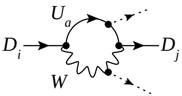

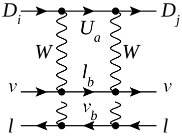

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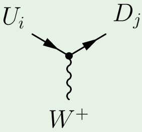

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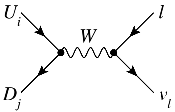

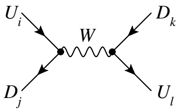

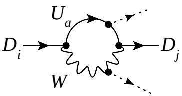

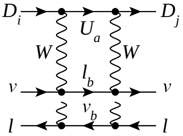

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$\mathcal{A} \sim G_F V_{ij}$

$\sim G_F V_{ij} V^*_{lk}$

$\sim G_F g \sum_a V_{ai} V^*_{aj} f(m_a)$

$\sim G_F g^2 \sum_{a,b} V_{ai} V^*_{aj} f(m_{a,b})$

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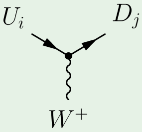

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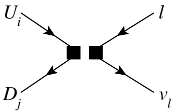

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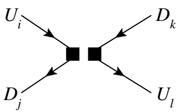

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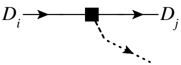

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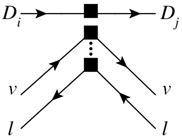

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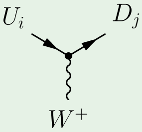


In SM FCNC-decays w.r.t. tree-decays are . . .

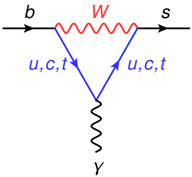
quantum fluctuations = loop-suppressed

- no suppression of contributions beyond SM (BSM) wrt SM itself
- indirect search for BSM signals

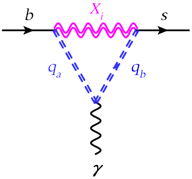
  $\Rightarrow$ additional contribution to **effective coupling *C***

BUT requires high precision, experimentally and theoretically !!!

SM



BSM



$$C(V_{ij}, m_a) \quad + \quad C(W_{ij}, m_X, m_q)$$

Fit of CKM matrix: Tree-level + $\Delta B = 2$ decays

$\Rightarrow$ fit of CKM-Parameters . . .

4 Wolfenstein parameters

$\lambda \sim 0.22$, A, ρ, η

$$V_{ij} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$\Rightarrow$ nowadays sophisticated fit: "combine and overconstrain" [CKMfitter, arXiv:1106.4041]

CKM	Process	Observables			Theoretical inputs
$\|V_{ud}\|$	$0^+ \to 0^+$ transitions	$\|V_{ud}\|_{\rm nucl}$	$= 0.97425 \pm 0.00022$	[6]	Nuclear matrix elements
$\|V_{us}\|$	$K \to \pi\ell\nu$	$\|V_{us}\|_{\rm semi} f_+(0)$	$= 0.2163 \pm 0.0005$	[7]	$f_+(0) = 0.9632 \pm 0.0028 \pm 0.0051$
	$K \to e\nu_e$	$\mathcal{B}(K \to e\nu_e)$	$= (1.584 \pm 0.0020) \cdot 10^{-5}$	[8]	$f_K = 156.3 \pm 0.3 \pm 1.9$ MeV
	$K \to \mu\nu_\mu$	$\mathcal{B}(K \to \mu\nu_\mu)$	$= 0.6347 \pm 0.0018$	[7]	
	$\tau \to K\nu_\tau$	$\mathcal{B}(\tau \to K\nu_\tau)$	$= 0.00696 \pm 0.00023$	[8]	
$\|V_{us}\|/\|V_{ud}\|$	$K \to \mu\nu / \pi \to \mu\nu$	$\frac{\mathcal{B}(K \to \mu\nu_\mu)}{\mathcal{B}(\pi \to \mu\nu_\mu)}$	$= (1.3344 \pm 0.0041) \cdot 10^{-2}$	[7]	$f_K/f_\pi = 1.205 \pm 0.001 \pm 0.010$
	$\tau \to K\nu / \tau \to \pi\nu$	$\frac{\mathcal{B}(\tau \to K\nu_\tau)}{\mathcal{B}(\tau \to \pi\nu_\tau)}$	$= (6.33 \pm 0.092) \cdot 10^{-2}$	[9]	
$\|V_{cd}\|$	$D \to \mu\nu$	$\mathcal{B}(D \to \mu\nu)$	$= (3.82 \pm 0.32 \pm 0.09) \cdot 10^{-4}$	[10]	$f_{D_s}/f_D = 1.186 \pm 0.005 \pm 0.010$
$\|V_{cs}\|$	$D_s \to \tau\nu$	$\mathcal{B}(D_s \to \tau\nu)$	$= (5.29 \pm 0.28) \cdot 10^{-2}$	[11]	$f_{D_s} = 251.3 \pm 1.2 \pm 4.5$ MeV
	$D_s \to \mu\nu$	$\mathcal{B}(D_s \to \mu\nu_\mu)$	$= (5.90 \pm 0.33) \cdot 10^{-3}$	[11]	
$\|V_{ub}\|$	semileptonic decays	$\|V_{ub}\|_{\rm semi}$	$= (3.92 \pm 0.09 \pm 0.45) \cdot 10^{-3}$	[11]	form factors, shape functions
	$B \to \tau\nu$	$\mathcal{B}(B \to \tau\nu)$	$= (1.68 \pm 0.31) \cdot 10^{-4}$	[4]	$f_{B_s} = 231 \pm 3 \pm 15$ MeV
					$f_{B_s}/f_B = 1.209 \pm 0.007 \pm 0.023$
$\|V_{cb}\|$	semileptonic decays	$\|V_{cb}\|_{\rm semi}$	$= (40.89 \pm 0.38 \pm 0.59) \cdot 10^{-3}$	[11]	form factors, OPE matrix elts
α	$B \to \pi\pi, \rho\pi, \rho\rho$	branching ratios, CP asymmetries		[11]	isospin symmetry
β	$B \to (c\bar{c})K$	$\sin(2\beta)_{[c\bar{c}]}$	$= 0.678 \pm 0.020$	[11]	
γ	$B \to D^{(*)}K^{(*)}$	inputs for the 3 methods		[11]	GGSZ, GLW, ADS methods
$V_{tq}^* V_{tq'}$	Δm_d	Δm_d	$= 0.507 \pm 0.005$ ps^{-1}	[11]	$\hat{B}_{B_s}/\hat{B}_{B_d} = 1.01 \pm 0.01 \pm 0.03$
	Δm_s	Δm_s	$= 17.77 \pm 0.12$ ps^{-1}	[12]	$\hat{B}_{B_s} = 1.28 \pm 0.02 \pm 0.03$
$V_{tq}^* V_{tq'}, V_{cq}^* V_{cq'}$	ϵ_K	$\|\epsilon_K\|$	$= (2.229 \pm 0.010) \cdot 10^{-3}$	[8]	$\hat{B}_K = 0.730 \pm 0.004 \pm 0.036$
					$\kappa_\epsilon = 0.940 \pm 0.013 \pm 0.023$

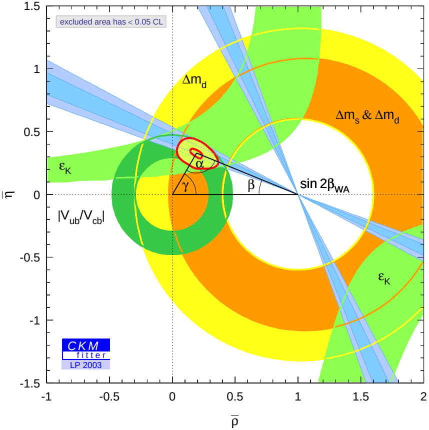
Fit of CKM matrix: Tree-level + $\Delta B = 2$ decays

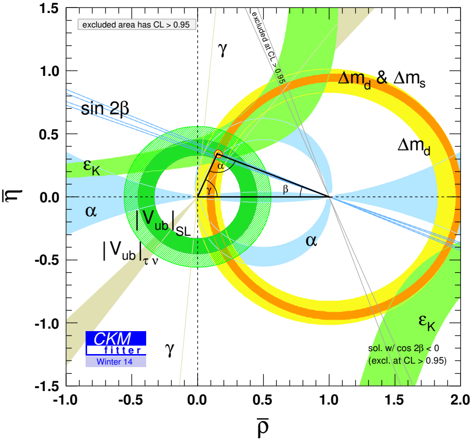
$\Rightarrow$ fit of CKM-Parameters ... 2003 $\rightarrow$ 2014

http://ckmfitter.in2p3.fr/ :

improved by *B*-factories, Tevatron, LHC

Unitarity: $V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0$





See also UTfit collaboration **http://www.utfit.org/UTfit/**

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Pursue similar global fit for $\Delta B = 1$ FCNC decays:

$b \to s\gamma$ and $b \to s\bar{\ell}\ell$

in combination with: quark masses, *B* form factors …

Rich phenomenology . . .

$b \to s + \gamma$

$B \to K^*\gamma \quad (B_s \to \phi\gamma)$

- Br
- time-dependent CP asy's: S, C, H
- iso-spin asymmetry Δ_{0-}

$B \to X_s\gamma$

- Br, dBr/dE_γ
- $A_{\rm CP}$ in $B \to X_s\gamma$ and $B \to X_{s+d}\gamma$

$B_s \to \gamma\gamma$

- Br
- $A_{\rm CP}$

$b \to s + \bar{\ell}\ell$

$B_s \to \bar{\ell}\ell$

- Br

$B \to K + \bar{\ell}\ell$

- $d^2Br/dq^2\, d\cos\theta_\ell \to dBr/dq^2$, $A_{\rm FB}$, F_H

$B \to K^*(\to K\pi) + \bar{\ell}\ell \quad (B_s \to \phi(\to \bar{K}K) + \bar{\ell}\ell)$

- $d^4Br/dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi$

12 angular observables $J^{(s,c)}_{1,\ldots,9}(q^2)$ + CP-conj.

$\to dBr/dq^2$, $A_{\rm FB}$, F_L, $A_T^{(2,3,4,{\rm re},{\rm im})}$, $H_T^{(1,2,3,4,5)}$, . . .

$B \to X_s + \bar{\ell}\ell$

- $d^2Br/dq^2\, d\cos\theta_\ell$, $A_{\rm FB}$, H_T (or H_L)

. . . in $b \to s + \{\gamma, \gamma\gamma, \bar{\ell}\ell\}$ FCNC's to test short-distance **effective couplings**:

C_i for $i = 7, (7')$

C_i for $i = 7, 9, 10, (7', 9', 10', \ldots)$

BUT need **non-perturbative hadronic quantities**: (complementarity of exclusive and inclusive)

Decay constants and LCDA's for $B_{d,s}$, K, K^*, ϕ, . . .

Form factors: $(B \to K) \to f_{+,T,0}$ and $(B \to K^*, B_s \to \phi) \to V, A_{0,1,2}, T_{1,2,3}$

Experimental number of events: $b \to s(d)\,\bar{\ell}\ell$

# of evts	BaBar	Belle	CDF	LHCb	CMS	ATLAS
	2012	2009	2011	2011 (+2012)	2011 (+2012)	2011
	471 M $\bar{B}B$	605 fb^{-1}	9.6 fb^{-1}	1 (+2) fb^{-1}	5 (+20) fb^{-1}	5 fb^{-1}
$B^0 \to K^{*0}\,\bar{\ell}\ell$	$137 \pm 44^\dagger$	$247 \pm 54^\dagger$	288 ± 20	2361 ± 56	415 ± 70	426 ± 94
$B^+ \to K^{*+}\bar{\ell}\ell$			24 ± 6	162 ± 16		
$B^+ \to K^+\bar{\ell}\ell$	$153 \pm 41^\dagger$	$162 \pm 38^\dagger$	319 ± 23	4746 ± 81	not yet	not yet
$B^0 \to K^0_S\,\bar{\ell}\ell$			32 ± 8	176 ± 17		
$B_s \to \phi\,\bar{\ell}\ell$			62 ± 9	174 ± 15		
$B_s \to \bar{\mu}\mu$				emerging	emerging	limit
$\Lambda_b \to \Lambda\,\bar{\ell}\ell$			51 ± 7	78 ± 12		
$B^+ \to \pi^+\,\bar{\ell}\ell$		limit		25 ± 7		
$B_d \to \bar{\mu}\mu$			limit	limit	limit	limit

- CP-averaged results
- J/ψ and ψ' q^2-regions vetoed
- † unknown mixture of B^0 and $B^\pm$
- $\ell = \mu$ for CDF, LHCb, CMS, ATLAS

Babar arXiv:1204.3933 + 1205.2201
Belle arXiv:0904.0770
CDF arXiv:1107.3753 + 1108.0695 + Public Note 10894
LHCb arXiv:1205.3422 + 1209.4284 + 1210.2645 + 1210.4492 + 1304.6325 + 1305.2168 + 1306.2577 + 1307.5024 + 1307.7595 + 1308.1340 + 1308.1707 + 1403.8044 + 1403.8045 + 1406.6482
CMS arXiv:1307.5025 + 1308.3409
ATLAS ATLAS-CONF-2013-038

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$B^0 \to K_S^0\,\bar{\ell}\ell$			32 ± 8	176 ± 17		
$B_s \to \phi\,\bar{\ell}\ell$			62 ± 9	174 ± 15		
$B_s \to \bar{\mu}\mu$				emerging	emerging	limit
$\Lambda_b \to \Lambda\,\bar{\ell}\ell$			51 ± 7	78 ± 12		
$B^+ \to \pi^+\,\bar{\ell}\ell$		limit		25 ± 7		
$B_d \to \bar{\mu}\mu$			limit	limit	limit	limit

Outlook / Prospects

- **Belle** reprocessed all data 711 fb^{-1} $\to$ no final analysis yet!
- **LHCb** $\sim$ 2 fb^{-1} from 2012 to be analysed and $\gtrsim$ 8 fb^{-1} by the end of 2018
- **ATLAS / CMS** $\sim$ 20 fb^{-1} from 2012 to be analysed
- **Belle II** expects about (10-15) K events $B \to K^*\bar{\ell}\ell$ ($\gtrsim$ 2020) [Bevan arXiv:1110.3901]

Effective Theory (EFT) of

$|\Delta B| = |\Delta S| = 1$ decays

B-Hadron decays are a Multi-scale problem ...

... with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in *B* restframe	$\gg$	QCD-bound state effects
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV		$M_B \approx 5$ GeV		$\Lambda_{\rm QCD} \approx 0.5$ GeV

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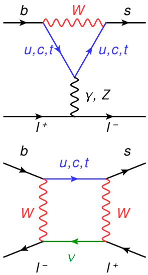
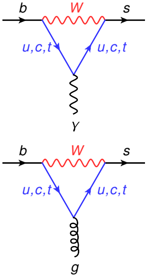
$$\mathcal{L}_{\rm eff} \sim G_F\, V_{\rm CKM} \times \Big[\sum_{9,10} C_i^{\ell\bar\ell}\, \mathcal{O}_i^{\ell\bar\ell} + \sum_{7\gamma,8g} C_i\, \mathcal{O}_i + \mathrm{CC} + (\text{QCD \& QED-peng}) \Big]$$

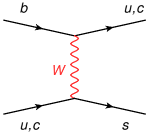
semi-leptonic



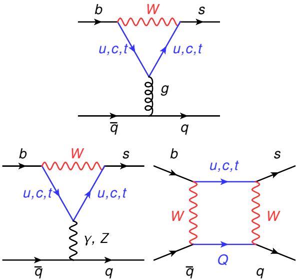
electro- & chromo-mgn



charged current



QCD & QED -penguin



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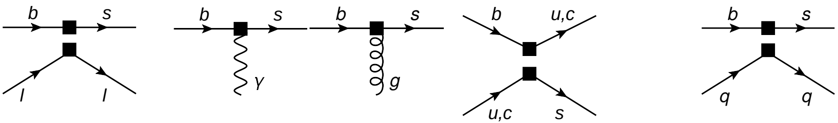
semi-leptonic | electro- & chromo-mgn | charged current | QCD & QED -penguin

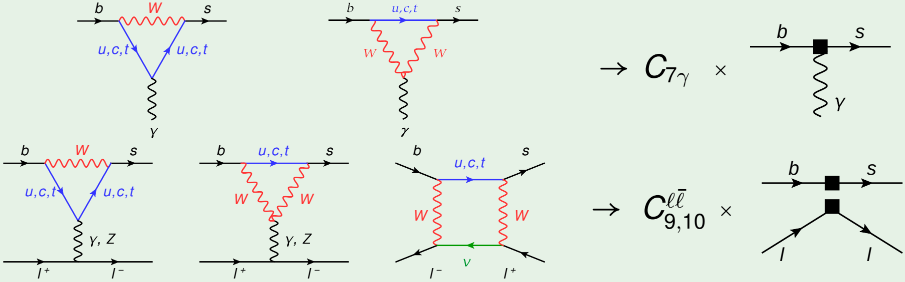


C_i = Wilson coefficients: contains short-dist. pmr's (heavy masses $M_t, \ldots$ – CKM factored out) and leading logarithmic QCD-corrections to all orders in α_s

$\Rightarrow$ in SM known up to next-to-next-to-leading order

$\mathcal{O}_i$ = higher-dim. operators: flavour-changing coupling of light quarks

Most important operators in the SM for $b \to s + (\gamma, \bar{\ell}\ell)$



$$\mathcal{O}_{7\gamma} \propto m_b[\bar{s}\,\sigma^{\mu\nu} P_R\, b]F_{\mu\nu}, \qquad \mathcal{O}_{9(10)}^{\ell\bar{\ell}} \propto [\bar{s}\,\gamma^{\mu} P_L\, b][\bar{\ell}\,\gamma_{\mu}(\gamma_5)\,\ell]$$

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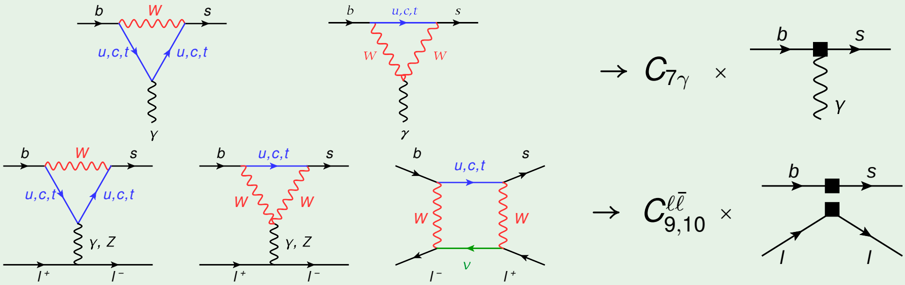

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and other contributions from

CC op's	$b \to s + \overline{U}U \;\; (U = u, c)$
QCD peng op's	$b \to s + \overline{Q}Q \;\; (Q = u, d, s, c, b)$
chromo-mgn op	$b \to s + gluon$

$\Rightarrow$ induce backgrounds

$$b \to s + (\overline{Q}Q) \to s + \bar{\ell}\ell$$

vetoed in exp's for $Q = c$: J/ψ and ψ'

Beyond the SM $b \to s + (\gamma, \bar{\ell}\ell)$ operators ...

... frequently considered in model-(in)dependent searches

SM' = χ-flipped SM analogues ($P_L \leftrightarrow P_R$)

$$\mathcal{O}_{7'\gamma} \propto m_b[\bar{s}\,\sigma_{\mu\nu} P_L\, b] F^{\mu\nu} \qquad \mathcal{O}_{9'(10')} \propto [\bar{s}\,\gamma^\mu P_R\, b][\bar{\ell}\,\gamma_\mu(\gamma_5)\,\ell]$$

S + P = scalar + pseudoscalar

$$\mathcal{O}_{S(S')} \propto [\bar{s}\,P_{R(L)}\,b][\bar{\ell}\,\ell] \qquad \mathcal{O}_{P(P')} \propto [\bar{s}\,P_{R(L)}\,b][\bar{\ell}\,\gamma_5\,\ell]$$

T + T5 = tensor

$$\mathcal{O}_T \propto [\bar{s}\,\sigma_{\mu\nu}\,b][\bar{\ell}\,\sigma^{\mu\nu}\,\ell] \qquad \mathcal{O}_{T5} \propto \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}[\bar{s}\,\sigma_{\mu\nu}\,b][\bar{\ell}\,\sigma_{\alpha\beta}\,\ell]$$

new Dirac-structures beyond SM:

SM'	=	right-handed currents
S + P	=	scalar-exchange & box-type diagrams
T + T5	=	box-type diagrams, Fierzed scalar tree exchange

Extension of EFT beyond the SM ...

$$\mathcal{L}_{\text{eff}}(\mu_b) = \mathcal{L}_{\text{QED}\times\text{QCD}}(u, d, s, c, b,\ e, \mu, \tau,\ ???)$$

$$+ \frac{4G_F}{\sqrt{2}} V_{\text{CKM}} \sum_{\text{SM}} (C_i + \Delta C_i)\mathcal{O}_i + \sum_{\text{NP}} C_j \mathcal{O}_j(???)$$

ΔC_i	=	NP contributions to SM C_i
$\sum_{\text{NP}} C_j \mathcal{O}_j$	=	NP operators (e.g. $C'_{7,9,10}$, $C^{(\prime)}_{S,P}$, ...)
???	=	additional light degrees of freedom ($\Leftarrow$ usually not pursued)

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model-dep. 1) decoupling of new heavy particles @ NP scale: $\mu_{NP} \gtrsim M_W$

2) RG-running to lower scale $\mu_b \sim m_b$ (potentially tower of EFT's)

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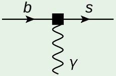
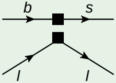
From EFT to observables

example exclusive $B \to K^* (\to K\pi)\bar{\ell}\ell$

Exclusive $B \to K^*(\to K\pi)\,\bar{\ell}\ell$ . . . using narrow width appr. & intermediate K^* on-shell

Hadronic amplitude $B \to K^*(\to K\pi)\,\bar{\ell}\ell$ neglecting 4-quark operators

$\mathcal{A}_\lambda = \langle K^*_\lambda |\quad C_7 \times$ 

$+ \quad C_{9,10} \times$ 

$|B\rangle$

$\mathcal{A}_\lambda$ = transversity amplitudes of K^* $(\lambda = \perp, \parallel, 0)$

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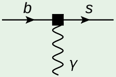
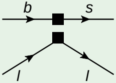


$+\; C_{9,10} \times$


$|B\rangle$

$\mathcal{A}_\lambda$ = transversity amplitudes of K^* $(\lambda = \perp, \parallel, 0)$

- "Naive factorisation" of leptonic and quark currents: $\mathcal{A}_\lambda \sim C_i\,[\bar{\ell}\,\Gamma_i'\,\ell] \otimes \langle K^* | \bar{s}\,\Gamma_i\, b | B\rangle$
- "just" requires $B \to K^*$ form factors (=FF): V, $A_{1,2}$, $T_{1,2,3}$ (A_0 contribution $\sim 2m_\ell/\sqrt{q^2}$)

$$A_\perp^{L,R} \simeq \sqrt{2\,\lambda}\left[(C_9 \mp C_{10})\frac{V}{M_B + M_{K^*}} + \frac{2\,m_b}{q^2} C_7\, T_1\right]$$

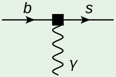
$$A_\parallel^{L,R} \simeq -\sqrt{2}\,(M_B^2 - M_{K^*}^2)\left[(C_9 \mp C_{10})\frac{A_1}{M_B - M_{K^*}} + \frac{2\,m_b}{q^2} C_7\, T_2\right]$$

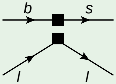
$$A_0^{L,R} \simeq -\frac{1}{2\,M_{K^*}\sqrt{q^2}}\left\{(C_9 \mp C_{10})\Big[\ldots A_1 + \ldots A_2\Big] + 2\,m_b C_7\Big[\ldots T_2 + \ldots T_3\Big]\right\}$$

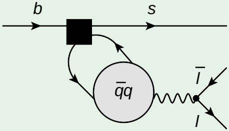
- FF's @ low q^2: light-cone sum rules [Ball/Zwicky hep-ph/0412079, Khodjamirian et al. arXiv:1006.4945]
- FF's @ high q^2: lattice calculations [Horgan/Liu/Meinel/Wingate arXiv:1310.3722, 1310.3887]

Exclusive $B \to K^*(\to K\pi)\,\bar{\ell}\ell$ … using narrow width appr. & intermediate K^* on-shell

Hadronic amplitude $B \to K^*(\to K\pi)\,\bar{\ell}\ell$ — including 4-quark operators

$\mathcal{A}_\lambda = \langle K^*_\lambda |\quad C_7 \times$ 

$+ \quad C_{9,10} \times$ 

$+ \sum_i C_i \times$ 

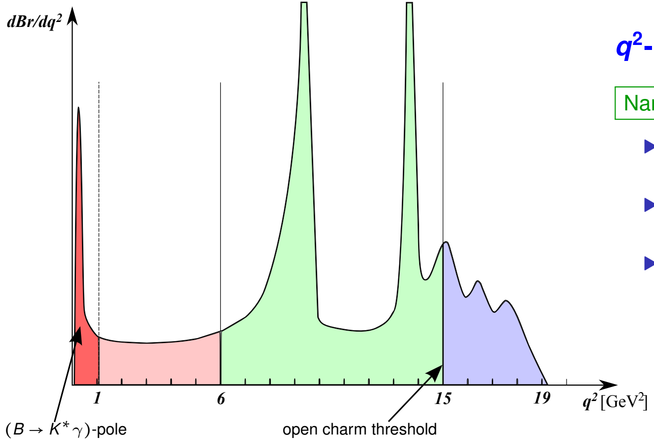
$|B\rangle$

… but 4-Quark operators and $\mathcal{O}_{8g}$ have to be included ⇒ no “naive factorisation” !!!

- current-current $b \to s + (\bar{u}u,\, \bar{c}c)$ — ($b \to s\,\bar{u}u$ suppressed by $V_{ub}V_{us}^*$)
- QCD-penguin operators $b \to s + \bar{q}q$ $(q = u, d, s, c, b)$ — (small Wilson coefficients)

⇒ large peaking background around certain $q^2 = (M_{J/\psi})^2,\, (M_{\psi'})^2$:

$B \to K^{(*)}(\bar{q}q) \to K^{(*)}\bar{\ell}\ell$



q^2-Regions in $B \to K^* \bar{\ell}\ell$

Narrow resonances

- dominated by charged-cur. (tree-level) op's
- not sensitive to new physics in $b \to s\bar{\ell}\ell$
- nonperturbative predictions via: dispersion relations + $B \to K^*(\bar{c}c)$ data

Large Recoil (low-q^2)

- very low-q^2 ($\lesssim 1$ GeV^2) dominated by $\mathcal{O}_7$
- low-q^2 ($[1, 6]$ GeV^2) dominated by $\mathcal{O}_{9,10}$
- 1) QCD factorization or SCET
  2) LCSR
  3) non-local OPE of $\bar{c}c$-tails

Low Recoil (high-q^2)

- dominated by $\mathcal{O}_{9,10}$
- local OPE (+ HQET) $\Rightarrow$ theory only for sufficiently large q^2-integrated obs's

EOS: Rare *B* decays

Global data analysis =

fit **“New Physics” parameters** combining
various observables of rare *B* decays

AND

account simultaneously for theory uncertainties by
inclusion of relevant (mostly nonperturbative) parameters
⇒ **“Nuisance” parameters**

USING

Bayesian inference to update knowledge on
New Physics & Nuisance parameters

⇒

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@ http://project.het.physik.tu-dortmund.de/eos/

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EOS collaboration

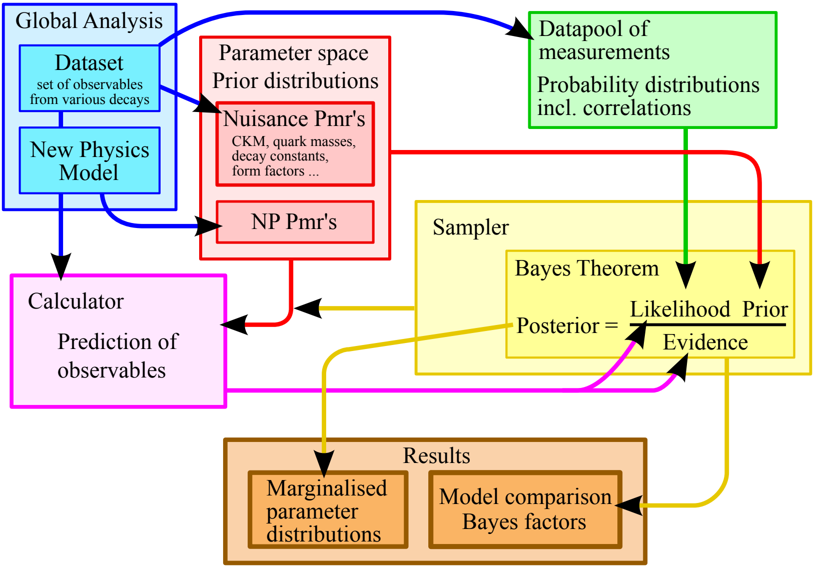
Danny van Dyk (University Siegen)
Frederik Beaujean (Universe Cluster - LMU Munich)
Christoph Bobeth (TU Munich)
Stephan Jahn (TU Munich)

Formerly: Christian Wacker

Contributors

LHCb: A. Shires (TU Dortmund)
Ch. Langenbruch and Th. Blake (U. Warwick)
K. Petridis (U. Bristol)
CDF: Hideki Miyake (Tsukuba U.)

EOS: Workflow of global data analysis . . .



Newly developed Sampler: Population Monte Carlo (PMC) initialized with Markov Chain samples

⇒ highly parallelizable ! [Beaujean/CB/van Dyk/Wacker arXiv:1205.1838, Beaujean/Caldwell arXiv:1304.7808]

EOS: Steering the fit

Fits are done with **EOS**-client program: **eos − scan − mc**

⇒ configured via command-line options → we use shell scripts

Example

fit Wilson coefficient C_{10} (real part, flat prior) from $Br(B_s \to \bar{\mu}\mu)$ of LHCb + CMS 2014, with nuisance parameters from CKM and B_s decay constant (gaussian priors with support of 3σ)

> eos − scan − mc

```
--global-option model WilsonScan \\
--global-option scan-mode cartesian \\
--constraint B^0_s->mu^+mu^-::BR@CMS-LHCb-2014 \\
--scan Re{c10} -1.0 7.0 --prior flat \\
--nuisance CKM::lambda 3 --prior gaussian 0.2247 0.2253 0.2259 \\
--nuisance CKM::... \\
--nuisance decay-constant::B_s 3 --prior gaussian 0.2232 0.2277 ...
```

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```

Parallelization

- threading on single multi-core machine possible
- parallelization of MCMC trivial (→ hierarchical clustering merges chains later on)
- parallelization of PMC highly dependent on queuing system of available cluster

  ⇒ achieved by multiple runs of **eos − scan − mc**

  ⇒ python script used for steering of PMC for

  1) sampling step, 2) update step of mixture density and 3) convergence check

EOS: Implemented observables $b \to s(\gamma, \bar{\ell}\ell)$

decay	observables	remarks
$B \to X_s\gamma$	$Br(E_\gamma)$, $\langle E \rangle_{1,2}$	@ NLO, E_γ photon energy cut 1st & 2nd photon energy moments
$B \to K^*\gamma$	Br, $\langle Br \rangle_{\rm CP}$ S, C, A_I	using QCDF, $\langle \cdot \rangle_{\rm CP}$ = CP-averaged CP-asym's and isospin asymmetry
$B_s \to \bar{\mu}\mu$	$Br(t=0)$, $\int dt\, Br(t)$ S, H, $\tau_{\rm eff}$	time-integ. Br @ NLO CP-asymmetries & eff. lifetime
$B \to X_s\bar{\ell}\ell$	Br	@ NNLO, low-q^2, q^2-diff. & integr.
$B \to K\bar{\ell}\ell$	Br, $A_{\rm CP}$, $A_{\rm FB}$, F_H $R_K = Br(\ell = \mu)/Br(\ell = e)$	@ low-q^2 QCDF, @ high-q^2 local OPE q^2-diff. & integr., also $\langle \cdot \rangle_{\rm CP}$
$B \to K^*\bar{\ell}\ell$	$d^4\Gamma/(dq^2\, d\phi\, d\cos\theta_\ell\, d\cos\theta_K)$ $J_{1s,1c,2s,2c,3,4,5,6s,6c,7,8,9}$ Br, F_L, F_T, $A_{\rm FB}$ $A_T^{(2,3,4,5,{\rm Re},{\rm Im})}$, $P'_{4,5,6}$ $H_T^{(1,2,3,4,5)}$, $a_{\rm CP}^{(1,2,3,{\rm mix})}$	$K^* \to K\pi$ on resonance @ low-q^2 QCDF, @ high-q^2 local OPE q^2-diff. & integr., also $\langle \cdot \rangle_{\rm CP}$ optimised observables @ low- and high-q^2

EOS:

Model-independent Fits

Recent “Global Fits” after EPS-HEP 2013 Conference

1) DGMV	=	Descotes-Genon/Matias/Virto	[arXiv:1307.5683 + 1311.3876]	χ^2-frequentist
2) AS-1 (-2)	=	Altmannshofer/Straub	[arXiv:1308.1501 (& 1411.3161)]	χ^2-fit
3) BBvD	=	Beaujean/CB/van Dyk	[arXiv:1310.2478v3]	Bayesian
4) HLMW	=	Horgan/Liu/Meinel/Wingate	[arXiv:1310.3887v3]	χ^2-fit

see also [Hurth/Mahmoudi arXiv:1312.5267, Hurth/Mahmoudi/Neshatpour arXiv:1410.4545]

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Theory predictions

@ low q^2: $B \to K^* \bar{\ell}\ell$, $B \to K\bar{\ell}\ell$, $B \to K^*\gamma$

DGMV, AS, BBvD: based on QCDF [Beneke/Feldmann/Seidel hep-ph/0106067 + 0412400]
(HLMW only uses high-q^2 data)

@ high q^2: $B \to K^* \bar{\ell}\ell$, $B \to K\bar{\ell}\ell$

DGMV, AS, BBvD, HLMW: based on local OPE
[Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

DGMV, AS-1, BBvD: LCSR $B \to K^*$ FF-results extrapolated from low q^2

HLMW, AS-2, BBvD: use lattice $B \to K^*$ FF predictions [HLMW arXiv:1310.3722]

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@ high q^2: $B \to K^* \bar{\ell}\ell$, $B \to K\bar{\ell}\ell$

DGMV, AS, BBvD, HLMW: based on local OPE

[Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

DGMV, AS-1, BBvD: LCSR $B \to K^*$ FF-results extrapolated from low q^2

HLMW, AS-2, BBvD: use lattice $B \to K^*$ FF predictions [HLMW arXiv:1310.3722]

Theory uncertainties

DGMV, AS, HLMW: combining theoretical and experimental uncertainties
$\Rightarrow$ included in likelihood

BBvD: most relevant parameters included in the fit as nuisance parameters

Which data is used?

† if P_2 is available then $A_{\rm FB}$ is not used: LHCb

decay	obs	DGMV	AS-1 (-2)	BBvD	HLMW
$B \to X_s\gamma$	Br	✓	✓	✓	
	A_{CP}		✓		
$B \to K^*\gamma$	Br			✓	
	$S(C)$	✓	✓	✓ (✓)	
	A_I	✓	(✓)		
$B_s \to \bar{\mu}\mu$	Br	✓	✓	✓	
$B \to X_s\bar{\ell}\ell$	Br	lo	lo+HI	lo	
$B \to K\bar{\ell}\ell$	Br		lo+HI (LO'+hi)	lo+HI	
$B \to K^*\bar{\ell}\ell$	Br		lo+HI (Lo+hi)	lo+HI	HI & hi
	F_L		lo+HI (Lo+hi)	lo+HI	HI & hi
	$A_{\rm FB}$	LO+HI	lo+HI (Lo+hi)	lo+HI†	HI & hi
	$P^{(\prime)}_{1,2,4,5,6}$	LO+HI		lo+HI†	
	P'_8	LO+HI			
	$S_{3,4,5}$		lo+HI (Lo+hi)		HI & hi
	A_9		lo+HI (Lo+hi)		
$B_s \to \phi\bar{\ell}\ell$	Br		(lo+hi)		HI & hi
	F_L, S_3		(lo+hi)		HI & hi

q^2 Binning

[GeV^2]	q^2-Bins
lo	[1, 6]
Lo	[< 2]
	[2, 4.3]
LO	[< 2]
	[2, 4.3]
	[4.3, 8.7]
hi	[> 16]
HI	[14.2, 16]
	[> 16]

DGMV: only LHCb data of $B \to K^*\bar{\ell}\ell$

AS-1, BBvD, HLMW: use all available data from Belle, Babar, CDF, LHCb, CMS, ATLAS

AS-2: exclude Belle, Babar if $\ell = e, \mu$

BBvD Current nuisance parameters . . .

A) . . . common parameters: CKM, quark masses, . . .

B) . . . describing q^2-dependence of form factors

- $B \to K$: $2\times \to$ prior from LCSR + Lattice
- $B \to K^*$: $6\times \to$ prior from 1) LCSR (NO Lattice)
  OR 2) LCSR + Lattice

C) . . . of naive parametrisation of subleading corrections

- $B \to K$: $2\times$ @ low and high q^2
- $B \to K^*$: $6\times$ @ low q^2 and $3\times$ @ high q^2

priors: about 15%$\sim \Lambda_{\rm QCD}/m_b$ of leading amplitude

. . . in total 28 nuisance parameters

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. . . in total 28 nuisance parameters

Model-independent New Physics scenarios

Fits in the SM

1) **SM** = only nuisance parameters

and model-independent scenarios

2) $\mathbf{SM_{7,9,10}}$ = $C_{7,9,10}^{\rm NP} \neq 0$
3) **SM+SM'** = $C_{7,9,10}^{\rm NP} \neq 0$ and $C_{7',9',10'} \neq 0$
4) $\mathbf{SM+SM'_{9,9'}}$ = $C_9^{\rm NP} \neq 0$ and $C_{9'} \neq 0$

Fitting nuisance parameters

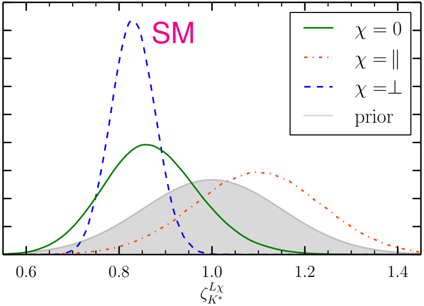
subleading corrections

$\Rightarrow$ in SM some subleading $B \to K^*$ corrections

$\sim -(15 - 20)\%$ for $\chi = \perp, 0$ @ low q^2

$\sim +10\%$ for $\chi = \parallel$

with gaussian priors of $1\sigma \sim \Lambda_{\rm QCD}/m_b \sim 15\%$



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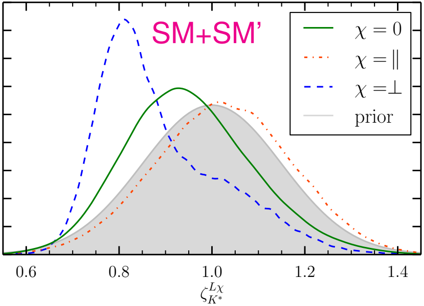
$\Rightarrow$ relaxed in SM+SM', except $\zeta_{K^*}^{L\perp}$



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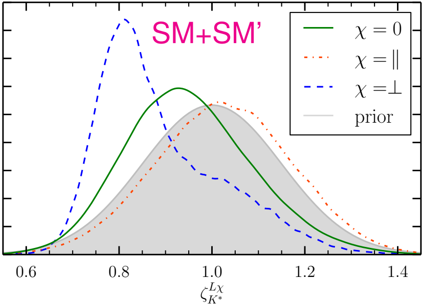

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$B \to K^*$ form factors

$$F(q^2) = \frac{F(0)}{1 - q^2/m^2_{B_s(J^P)}} \left[1 + b_1^F \times \ldots\right]$$

FF-parameterisation: $F(0)$, b_1^F
based on z-parameterisation

- data yields similar posterior FF parameters in $\mathrm{SM}_{7,9,10}$ & SM+SM'
- lattice prior uncertainty comparable to posterior uncertainty from data

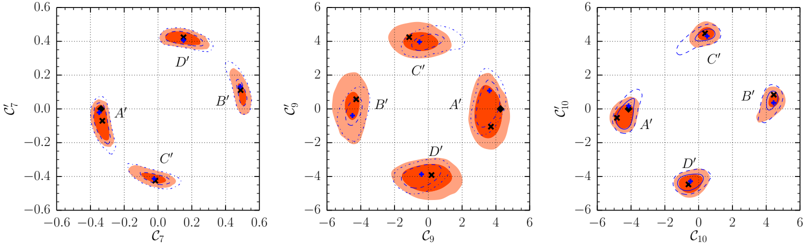
	no $B \to K^*$ lattice		with $B \to K^*$ lattice	
	prior	SM	prior	SM
$V(0)$	$0.35^{+0.14}_{-0.09}$	$0.40^{+0.03}_{-0.03}$	$0.36^{+0.03}_{-0.03}$	$0.38^{+0.03}_{-0.02}$
$A_1(0)$	$0.28^{+0.08}_{-0.07}$	$0.24^{+0.03}_{-0.02}$	$0.28^{+0.04}_{-0.03}$	$0.26^{+0.03}_{-0.02}$
$A_2(0)$	$0.24^{+0.13}_{-0.07}$	$0.23^{+0.04}_{-0.04}$	$0.28^{+0.05}_{-0.05}$	$0.25^{+0.04}_{-0.03}$

LCSR $B \to K^*$ FF's [Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]

lattice $B \to K^*$ FF's [Horgan/Liu/Meinel/Wingate arXiv:1310.3722]

Fitting effective couplings

SM+SM'



- 4 solutions with posterior masses: $A' = 37\%$, $B' = 14\%$, $C' = 15\%$, $D' = 34\%$
  with lattice $B \to K^*$ FF's: $A' = 35\%$, $B' = 16\%$, $C' = 17\%$, $D' = 32\%$
- largest deviation in 2D-plane $(C_9 - C_{7'})$ at 1.6σ

All scenarios:

inclusion of lattice $B \to K^*$ yields only minor changes in $\mathcal{C}_i(\mu = 4.2$ GeV$)$

$\Rightarrow$ largest effect on $\mathcal{C}_9$

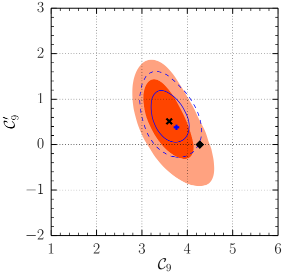
SM+SM'$_{9,9'}$

SM at

1.4σ without

2.0σ with

$B \to K^*$ FF's



red/blue = without/with $B \to K^*$ lattice FF's, $(\blacklozenge)$ = SM, $(\times)$ = best fit point

Goodness of fit

$\Rightarrow$ In SM: 6 measurements (out of 92) with pull values $> 2\sigma$ @ best fit point:

Belle	:	$\langle Br \rangle_{[16,19]}$	$\rightarrow +2.6\sigma$			
BaBar	:	$\langle F_L \rangle_{[1,6]}$	$\rightarrow -3.4\sigma$			
LHCb	:	$\langle P'_4 \rangle_{[14,16]}$	$\rightarrow -2.4\sigma$	$\langle P'_5 \rangle_{[1,6]}$	$\rightarrow +2.3\sigma$	not yet published
ATLAS	:	$\langle A_{\rm FB} \rangle_{[16,19]}$	$\rightarrow +2.1\sigma$	$\langle F_L \rangle_{[1,6]}$	$\rightarrow -2.5\sigma$	

SM p values @ best fit point: 0.12 (and 0.06 with lattice $B \to K^*$ FF's)

excluding $\langle F_L \rangle_{[1,6]}$ from BaBar and ATLAS: 0.63 (and 0.55 with lattice $B \to K^*$ FF's)

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Model comparison of models M_1 and M_2 with priors $P(M_i)$ ($\leftarrow$ unknown!)

$$\frac{P(M_1|D)}{P(M_2|D)} = B(D|M_1, M_2)\frac{P(M_1)}{P(M_2)} \qquad \text{Bayes factor: } B(D|M_1, M_2) \equiv \frac{P(D|M_1)}{P(D|M_2)}$$

!!! Models with more parameters are disfavored by larger prior volume,
unless they improve the fit substantially

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ATLAS	:	$\langle A_{\mathrm{FB}} \rangle_{[16,19]}$	$\rightarrow$	$+2.1\sigma$	$\langle F_L \rangle_{[1,6]}$	$\rightarrow$	-2.5σ

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$B(D\|M_1, M_2)^\dagger$	$\mathrm{SM}_{7,9,10}$: SM	SM+SM' : SM	SM+SM'$_{9,9'}$: SM
no lattice FF's	1 : 48	1 : 401	1 : 3
with lattice FF's	1 : 43	1 : 148	1 : 1

$\delta C_{7(')} \in [-0.2, 0.2]$

$\delta C_{9('),10(')} \in [-2, 2]$

† H. Jeffreys interpretation of $B(D|M_1, M_2)$ as strength of evidence in favour of M_2:

1:3 < barely worth mentioning, 1:10 < substantial, 1:30 < strong, 1:100 < very strong, > 1:100 decisive.

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!!! Models with more parameters are disfavored by larger prior volume, unless they improve the fit substantially

SM wins, SM+SM'$_{9,9'}$ still competitive

$\Rightarrow$ better prior (= theoretical control) over subleading corrections needed

$\Rightarrow$ waiting eagerly for LHCb update with 3 fb^{-1}, hopefully Moriond 2015

$\Rightarrow$ updated analysis from BaBar, ATLAS, Belle would be also welcome

Summary & Outlook

Summary: EOS & rare B decays

EOS = HEP Flavour tool maintained by EOS collaboration

@ **http://project.het.physik.tu-dortmund.de/eos/**

- Bayesian inference analysis tool
- highly parallelizable sampling algorithm (MCMC + HC + PMC) for multi-modal target functions in high-dimensional parameter space
- theory uncertainties included via marginalisation of according nuisance parameters
- provides implementation of
  - $|\Delta B| = 1$ SM Wilson coefficients at NNLO
  - several parameterisations of $B_q \to (P, V)$ form factors and lattice priors
  - model-independent scenario of complete set of $|\Delta B| = |\Delta S| = 1$ Wilson coefficients
  - observables of exclusive decays: $B_s \to \bar{\mu}\mu$, $B \to K\bar{\ell}\ell$, $B \to K^*\bar{\ell}\ell$
  - observables of inclusive decays: $B \to X_s\gamma$, $B \to X_s\bar{\ell}\ell$
  - observables of exclusive decays: $B \to \pi\ell\bar{\nu}$
- large data pool of recent experimental results

$\Rightarrow$ successful global model-independent fit of rare B decays and model comparison

[Beaujean/CB/van Dyk arXiv:1310.2478v3]

EOS: Outlook

Package organisaton:

- split off sampling (statistics) from implementation of physics (observables)
  $\Rightarrow$ keep physics in C++ and provide interface to statistics package

Sampling:

- provide new algorithm using Variational Bayes (to replace hierarchical clustering)

  $\Rightarrow$ already available as **pypmc** (python) [Beaujean/Jahn https://github.com/fredRos/pypmc]
  $\Rightarrow$ interface to **EOS** under development [Beaujean/CB/Jahn]

User:

- User manual
- Simple plotting tool (python)
- GUI for steering simple fits (python)

Physics:

- optimise performance of existing implementations, add further corrections
- extend inclusive $|\Delta B| = 1$: A) NNLO $b \to s\gamma$ and B) semi-inclusive $b \to s\bar{\ell}\ell$
  $\Rightarrow$ combination of inclusive $b \to s(\gamma, \bar{\ell}\ell)$ with $b \to c\ell\bar{\nu}$ for inclusion of m_b and V_{cb}
- exclusive and inclusive $b \to s\bar{\nu}\nu$ [see talk Christoph Niehoff for physics case]
- $|\Delta B| = 2$ (mixing) and $|\Delta B| = |\Delta D| = 1$ observables
- charmless hadronic $B \to M_1 M_2$ decays (in QCDF)
- Kaon physics: rare $|\Delta S| = |\Delta D| = 1$ observables
- new physics models for model-dependent fits (2HDM, MSSM, ...)
- event generator for rare decays

Rare $b \to s + (\gamma, \bar{\ell}\ell)$ decays and Belle II

Inclusive decays $B \to X_s\gamma$ and $B \to X_s\bar{\ell}\ell$ are very important cross check

- because theoretical predictions involve completely different hadronic quantities than exclusive decays (heavy quark expansion, shape functions, etc.)
- $Br(B \to X_s\gamma) \propto |C_7(\mu_b)|^2$ provides most stringent bound
- $B \to X_c\ell\bar{\nu}$ provides control on correlation of $m_b(m_b)$ and V_{cb}, which enter $B \to X_s\gamma$

Exclusive decays

Don't be discouraged just because LHCb measures $B^0 \to K^{*0}\bar{\mu}\mu$ and $B^+ \to K^+\bar{\mu}\mu$ with "infinite" precision!

Is there a serious study of experimental reach, efficiencies etc. at Belle II?

- should try to check LHCb, and measure iso-spin partner modes
- what about $B \to K^{(*)}\bar{e}e$?
- provide bounds on 1) $B \to K^{(*)}\bar{\tau}\tau$ and 2) LFV $B_{d,s} \to \bar{\ell}_a\ell_b$ and $B \to K^{(*)}\bar{\ell}_a\ell_b$ for $a \neq b$
- try to measure $B \to K^{(*)}\bar{\nu}\nu$

LHCb might be well systematics-limited, because can not measure absolute rates

$\Rightarrow$ normalisation modes – like $B \to J/\psi + K^{(*)}$ – come from B-factories

$\Rightarrow$ Belle II has to improve them to make the most out of LHCb data!

Backup Slides

EOS: Sampling algorithm in 3 steps: MCMC + HC + PMC

1) Markov Chain pre-run (MCMC)

Multiple MC's run (in parallel) using Metropolis-Hastings to explore parameter space

- chains are started at random or drawn from prior positions in parameter space
- number of chains must be optimised by user
- parallelization is limited to parallel run of chains
  ⇒ a chain itself can not be parallelized due to serial nature of Metropolis-Hastings

Advantage: allows very efficient localisation and exploration of local modes

Problem: in multi-modal target density MC's usually trapped in local modes

⇒ MC's are not sufficiently mixed to be combined to single MC
⇒ criteria for mixing: Gelman-Rubin R-value

Disadvantage: no straightforward calculation of "evidence" for model comparison

EOS: Sampling algorithm in 3 steps: MCMC + HC + PMC

2) Hierarchical clustering (HC)

Transform MC's into mixture density of multi-variate gaussian functions
as initialisation of importance sampling PMC

- group MC chains using R-value (should correspond to local modes)
- split chains into sub-chains (patch) and generate components from their samples (component = multi-variate gaussian)
- use hierarchical clustering [Goldberger/Roweis Adv.Neur.Info.Proc.Syst. 17 (2004) 505] to combine components that are "redundant" based on Kullback-Leibler divergence

Advantage: allows to eliminate redundant components and reduce their number

Disadvantage: user needs to determine the final number of components (our rule of thumb: should be at least as large as dimension of parameter space)

$\Rightarrow$ "Variational Bayes" automatically determines number of relevant components

EOS: Sampling algorithm in 3 steps: MCMC + HC + PMC

3) Importance sampling via Population Monte Carlo (PMC)

- initialised with mixture density determined in MCMC + HC
  - ⇒ all components have equal weight
    (balance effect of unequal number of chains in local modes)
  - ⇒ can replace (all) gaussian components by student-t
    (with optional choice of fixed degrees of freedom → heavier tails)

- PMC algorithm proceeds iteratively
  1) draw samples from current mixture density
     (number of samples user choice, min. number of samples per component required)
  2) calculate new weights of components based on PMC algorithm

     [Cappé/Douc/Guillin/Martin/Robert arXiv: 0710.4242]

     [Wraith/Kilbinger/Benabed/Cappé/Cardoso/Fort/Prunet/Robert arXiv: 0903.0837]
  3) check convergence of “perplexity” and “effective sample size”

- draw larger set of samples in final step

Theory uncertainties in Global Fits

Parameters of interest

$\vec{\theta} = C_i$ (Wilson coeff's)

Theory uncertainties in Global Fits

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$\vec{\theta} = C_i$ (Wilson coeff's)

Nuisance parameters

$\vec{\nu}$

1) process-specific

   form factors & decay const's,
   LCDA pmr's,
   sub-leading Λ/m_b,
   renormalization scales: $\mu_{b,0}$

2) general

   quark masses, CKM, . . .

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Observables

1) observables

$$O(\vec{\theta}, \vec{\nu})$$

depend usually on sub-set of $\vec{\theta}$ and $\vec{\nu}$

2) experimental data for each observable

$$\text{pdf}(O = o)$$

$\Rightarrow$ probability distribution of values o

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Fit strategies: 1) Put theory uncertainties in likelihood:

$$\chi^2 = \sum \frac{(O_{\text{ex}} - O_{\text{th}})^2}{\sigma_{\text{ex}}^2 + \sigma_{\text{th}}^2}$$

- sample $\vec{\theta}$-space (grid, Markov Chain, importance sampling...)
- theory uncertainties of O_i at each $(\vec{\theta})_i$: vary $\vec{\nu}$ within some ranges $\Rightarrow \sigma_{\text{th}}(O[(\vec{\theta})_i])$
- use Frequentist or Bayesian method $\Rightarrow$ 68 & 95 % (CL or CR) regions of $\vec{\theta}$

Theory uncertainties in Global Fits

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Fit strategies: 2) Fit also nuisance parameters:

- sample $(\vec{\theta} \times \vec{\nu})$-space (grid, Markov Chain, importance sampling...)
- accounts for theory uncertainties by fitting also $(\vec{\nu})_i$
- use Frequentist or Bayesian method $\Rightarrow$ 68 & 95 % (CL or CR) regions of $\vec{\theta}$ and $\vec{\nu}$

Angular analysis of $\overline{B} \to \overline{K}^* [\to \overline{K}\pi] + \bar{\ell}\ell$

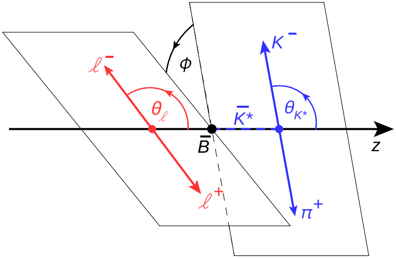
4-body decay with on-shell $\overline{K}^*$ (vector)

1) $q^2 = m^2_{\bar{\ell}\ell} = (p_\ell + p_{\bar{\ell}})^2 = (p_{\bar{B}} - p_{\bar{K}^*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle (\vec{p}_{\bar{B}}, \vec{p}_\ell)$ in $(\bar{\ell}\ell)$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle (\vec{p}_{\bar{B}}, \vec{p}_{\bar{K}})$ in $(\bar{K}\pi)$ – c.m. system

4) $\phi \angle (\vec{p}_{\bar{K}} \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



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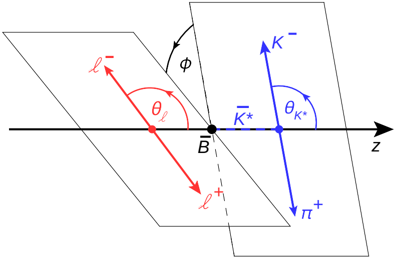
4) $\phi \angle (\vec{p}_{\bar{K}} \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



$J_i(q^2)$ = "Angular Observables"

$$\frac{32\pi}{9} \frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K + (J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_\ell$$

$$+J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$

$$+(J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi$$

$$+J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$

Angular analysis of $\bar{B} \to \bar{K}^* [\to \bar{K}\pi] + \bar{\ell}\ell$

4-body decay with on-shell $\bar{K}^*$ (vector)

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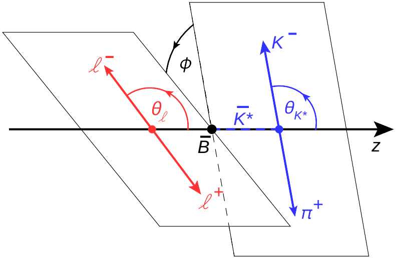

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$$+J_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + J_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi$$

⇒ "$2 \times (12 + 12) = 48$" if measured separately: A) decay + CP-conj and B) for $\ell = e, \mu$

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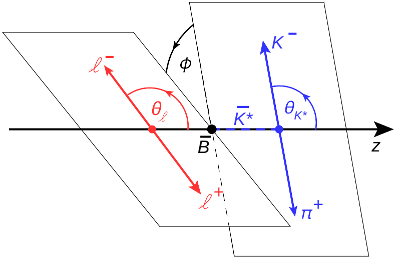

⇒ CP-averaged and CP-asymmetric angular observables

$$S_i = \frac{J_i + \bar{J}_i}{\Gamma + \bar{\Gamma}}, \qquad A_i = \frac{J_i - \bar{J}_i}{\Gamma + \bar{\Gamma}},$$

[Krüger/Sehgal/Sinha/Sinha hep-ph/9907386]
[Altmannshofer et al. arXiv:0811.1214]

CP-conj. decay $B^0 \to K^{*0}(\to K^+\pi^-)\ell^+\ell^-$: $d^4\bar{\Gamma}$ from $d^4\Gamma$ by replacing

$$\text{CP-even} : \quad J_{1,2,3,4,7} \longrightarrow +\bar{J}_{1,2,3,4,7}[\delta_W \to -\delta_W]$$

$$\text{CP-odd} : \quad J_{5,6,8,9} \longrightarrow -\bar{J}_{5,6,8,9}[\delta_W \to -\delta_W]$$

with weak phases δ_W conjugated

Angular observables & form factor (=FF) relations

$$J_i(q^2) \sim \{\text{Re}, \text{Im}\}\left[A_m^{L,R}\left(A_n^{L,R}\right)^*\right]$$

$$\sim \sum_a (C_a F_a) \sum_b (C_b F_b)^*$$

$A_m^{L,R} \ldots K^*$-transversity amplitudes $m = \perp, \parallel, 0$

$C_a \ldots$ short-distance coefficients

$F_a \ldots$ FF's

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simplify when using FF relations:

low K^* recoil limit: $E_{K^*} \sim M_{K^*} \sim \Lambda_{\text{QCD}}$ [Isgur/Wise PLB232 (1989) 113, PLB237 (1990) 527]

$$T_1 \approx V, \qquad T_2 \approx A_1, \qquad T_3 \approx A_2 \frac{M_B^2}{q^2}$$

large K^* recoil limit: $E_{K^*} \sim M_B$ [Charles et al. hep-ph/9812358, Beneke/Feldmann hep-ph/0008255]

$$\xi_\perp \equiv \frac{M_B}{M_B + M_{K^*}} V \approx \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 \approx T_1 \approx \frac{M_B}{2E_{K^*}} T_2$$

$$\xi_\parallel \equiv \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 - \frac{M_B - M_{K^*}}{M_{K^*}} A_2 \approx \frac{M_B}{2E_{K^*}} T_2 - T_3$$

"Optimized observables" in $B \to K^* \bar{\ell}\ell$

Idea: reduce form factor (=FF) sensitivity by combination (usually ratios) of angular obs's J_i

⇒ guided by large energy limit @ low-q^2 and Isgur-Wise @ high-q^2 FF-relations

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@ low q^2 = large recoil

$$A_T^{(2)} = P_1 = \frac{J_3}{2\,J_{2s}}, \qquad A_T^{(\mathrm{re})} = 2\,P_2 = \frac{J_{6s}}{4\,J_{2s}}, \qquad A_T^{(\mathrm{im})} = -2\,P_3 = \frac{J_9}{2\,J_{2s}},$$

$$P_4' = \frac{J_4}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_5' = \frac{J_5/2}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_6' = \frac{-J_7/2}{\sqrt{-J_{2c}J_{2s}}}, \qquad P_8' = \frac{-J_8}{\sqrt{-J_{2c}J_{2s}}},$$

$$A_T^{(3)} = \sqrt{\frac{(2\,J_4)^2 + J_7^2}{-2\,J_{2c}\,(2\,J_{2s} + J_3)}}, \qquad A_T^{(4)} = \sqrt{\frac{J_5^2 + (2\,J_8)^2}{(2\,J_4)^2 + J_7^2}}$$

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571]

[Becirevic/Schneider arXiv:1106.3283]

[Matias/Mescia/Ramon/Virto arXiv:1202.4266]

[Descotes-Genon/Matias/Ramon/Virto arXiv:1207.2753]

“Optimized observables” in $B \to K^* \bar{\ell}\ell$

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@ high q^2 = low recoil

$$H_T^{(1)} = P_4 = \frac{\sqrt{2} J_4}{\sqrt{-J_{2c}(2J_{2s} - J_3)}},$$

$$H_T^{(2)} = P_5 = \frac{J_5/\sqrt{2}}{\sqrt{-J_{2c}(2J_{2s} + J_3)}}, \qquad H_T^{(3)} = \frac{J_{6s}/2}{\sqrt{(2J_{2s})^2 - (J_3)^2}},$$

$$H_T^{(4)} = Q = \frac{\sqrt{2} J_8}{\sqrt{-J_{2c}(2J_{2s} + J_3)}}, \qquad H_T^{(5)} = \frac{-J_9}{\sqrt{(2J_{2s})^2 - (J_3)^2}},$$

$$\frac{A_9}{A_{\rm FB}} = \frac{J_9}{J_{6s}}, \qquad \text{and} \qquad \frac{J_8}{J_5}$$

[CB/Hiller/van Dyk arXiv:1006.5013]
[Matias/Mescia/Ramon/Virto arXiv:1202.4266]
[CB/Hiller/van Dyk arXiv:1212.2321]

Low-q^2 = Large Recoil: $E_{K^*} \sim m_b$

$\Rightarrow$ energetic "light" K^*, allows to calculate hard spectator scattering (HS) and weak annihilation (WA) in expansion in $\Lambda_{\rm QCD}/E_{K^*}$ and perturbatively in α_s

QCD Factorisation (QCDF)

[Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]

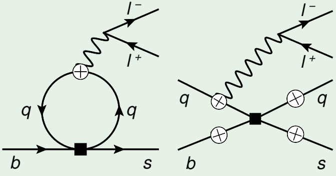
= (large recoil + heavy quark) limit (also Soft-Collinear Effective Theory = SCET)

$$\left\langle \bar{\ell}\ell K_a^* \,\middle|\, H_{\rm eff}^{(i)} \,\middle|\, B \right\rangle \sim$$

$$C_a^{(i)} \times \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + \mathcal{O}(\Lambda_{\rm QCD}/m_b)$$

$C_a^{(i)}$, $T_a^{(i)}$: perturbative kernels in α_s ($a = \perp, \parallel$, $i = u, t$)

ϕ_B, ϕ_{a,K^*} : B– and K_a^*–distribution amplitudes



- $C_a^{(i)}$ corrections ~ universal form factors ξ_a
- $T_a^{(i)}$ HS and WA contributions - numerically small in most observables
- breaks down at subleading order in $1/m_b$ → endpoint divergences

[Feldmann/Matias hep-ph/0212158]

$\Rightarrow$ may be large for some observables, especially optimised observables

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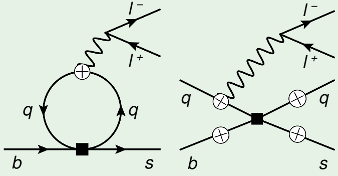

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  [Feldmann/Matias hep-ph/0212158]

  $\Rightarrow$ may be large for some observables, especially optimised observables

$\Rightarrow$ sub-leading soft gluon effects beyond QCDF from LCSR’s

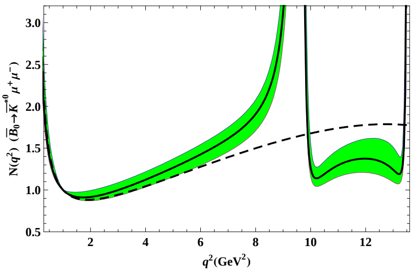
[Ball/Jones/Zwicky hep-ph/0612081, Dimou/Lyon/Zwicky arXiv:1212.2242, Lyon/Zwicky arXiv:1305.4797]

$\bar{c}c$-Resonances

@ low q^2 $\Rightarrow$ in general non-perturbative, $B \to K^* J/\psi (\to K^* \bar{\ell}\ell)$ colour-suppressed

- $-4m_c^2 \le q^2 \le 2\ \text{GeV}^2 \ll 4m_c^2$: non-local OPE near light-cone including soft-gluon emission
  $\Rightarrow$ matrix elmnt. via LCSR with B-meson DA's and light-meson interpolating current
  [Khodjamirian/Mannel/Offen hep-ph/0504091 & 0611193]
- $B \to K^{(*)}$ form factors also via same LCSR
- $q^2 \gtrsim 4\ \text{GeV}^2$: hadronic dispersion relation using measured $B \to K^{(*)} + (J/\psi, \psi')$
  $\to$ some modelling of spectral density
- matching both regions: destructive interference between J/ψ and ψ' contributions
- affects rate up to (15-20) % for $1 \lesssim q^2 \lesssim 6\ \text{GeV}^2$

[Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]



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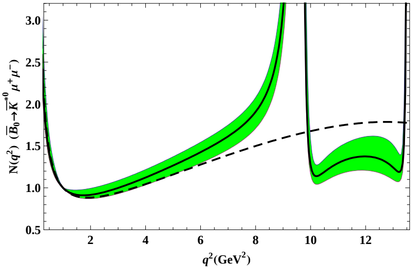
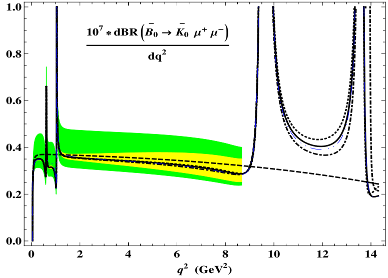




Extended to include light resonances $q = u, d, s$

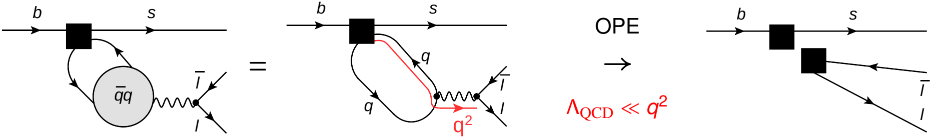
for $B \to K\bar{\ell}\ell$ [Khodjamirian/Mannel/Wang arXiv:1211.0234]

- non-local OPE done completely below hadronic threshold $q^2 < 0$

$\bar{c}c$-Resonances

@high q^2 [Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118]

Hard momentum transfer ($q^2 \sim M_B^2$) through $(\bar{q}q) \to \bar{\ell}\ell$ allows local OPE



$$\mathcal{A}[B \to K^* \,\bar{\ell}\ell] \sim \frac{8\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x} \langle K^* | T\{\mathcal{L}^{\rm eff}(0),\, j_\mu^{\rm em}(x)\} | B\rangle\, [\bar{\ell}\gamma^\mu \ell]$$

$$= \left(\sum_a \mathcal{C}_{3a} \mathcal{Q}_{3a}^\mu + \frac{m_s}{m_b} \times \text{dim-4} + \sum_b \mathcal{C}_{5b} \mathcal{Q}_{5b}^\mu + \mathcal{O}(dim > 5) \right) [\bar{\ell}\gamma_\mu \ell]$$

$dim = 3$ usual $B \to K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$, also α_s matching corrections known

$dim = 5$ suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2$ %, explicite estimate @ $q^2 = 15$ GeV^2: $< 1\%$

beyond OPE duality violating effects [Beylich/Buchalla/Feldmann arXiv:1101.5118]

- based on Shifman model for c-quark correlator + fit to recent BES data
- ± 2 % for integrated rate $q^2 > 15$ GeV^2

High-q^2 $\bar{c}c$-resonances in $B^+ \to K^+\bar{\mu}\mu$

[Lyon/Zwicky arXiv:1406.0566]

factorization assumption for $B \to K + \Psi(nS)(\to \bar{\ell}\ell)$:

$$\langle \Psi(nS)\, K|(\bar{c}\Gamma c)(\bar{s}\Gamma' b)|B\rangle \approx \langle \Psi(nS)|\bar{c}\Gamma c|0\rangle \otimes \langle K|\bar{s}\Gamma' b|B\rangle + \ldots$$ nonfactorisable

+ dispersion relations with BES II $\bar{e}e \to \bar{q}q$ data

+ comparison with LHCb 3 fb^{-1} of $B^+ \to K^+\bar{\mu}\mu$ @ high-q^2

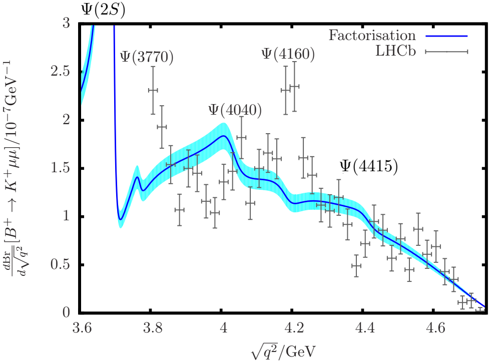
- factorization "badly fails" differentially in q^2
  - $\Rightarrow$ not unexpected, well-known from $B \to K\Psi(nS)$
  - $\Rightarrow$ "fudge factor" $\neq 1$
- does it invalidate the OPE ???

  this requires q^2-integration !!!
- investigate other $B \to M\bar{\ell}\ell$

  $M = K^*$ at LHCb

  $M = X_s$ (inclusive) at Belle II

  + including J/ψ and ψ'



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+ dispersion relations with BES II $\bar{e}e \to \bar{q}q$ data

+ comparison with LHCb 3 fb^{-1} of $B^+ \to K^+ \bar{\mu}\mu$ @ high-q^2

- a) no "fudge factor": $p = 0\%$

  various "generalisations of factorisable contributions"

  b) fit "fudge factor" = −2.6: $p = 1.5\%$

  c), d) fit rel. factors of $\Psi(nS)$: $p = 12\%$ and $p = 20\%$

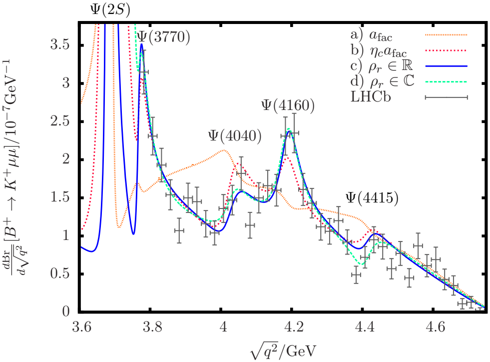
  ⇒ improve the combined fit of BES II and LHCb considerably

  (BES II data alone: $p = 44\%$)

- BUT can these parametrisations capture all features of non fact. contr.: Wilson coeffs. & q^2 ???
- can't be explained with NP in C_9

  ⇒ can ease tension in P_5'

  ⇒ NP in $b \to s\,\bar{c}c$?!



Subleading corrections to TransAmp's

$\lambda = \Lambda_{\mathrm{QCD}}/m_b \sim 0.15$

Low hadronic recoil

$$A_i^{L,R} \sim C^{L,R} \times f_i \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$)

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1 + \hat{M}_{K^*}} V, \qquad f_\parallel = \sqrt{2}\,(1 + \hat{M}_{K^*})\, A_1, \qquad f_0 = \frac{(1 - \hat{s} - \hat{M}_{K^*}^2)(1 + \hat{M}_{K^*})^2 A_1 - \hat{\lambda}\, A_2}{2\,\hat{M}_{K^*}(1 + \hat{M}_{K^*})\sqrt{\hat{s}}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Subleading corrections to TransAmp's

$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil

FF symmetry breaking

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s)$$

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1 SD-coefficient $C^{L,R}$ and 3 FF's f_i $(i = \perp, \parallel, 0)$

$C_7^{\rm SM} \approx -0.3,\ C_9^{\rm SM} \approx 4.2,\ C_{10}^{\rm SM} \approx -4.2$

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K^*}} V, \qquad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K^*})\,A_1, \qquad f_0 = \frac{(1-\hat{s}-\hat{M}_{K^*}^2)(1+\hat{M}_{K^*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K^*}(1+\hat{M}_{K^*})\sqrt{\hat{s}}}$$

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Subleading corrections to TransAmp's

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Low hadronic recoil

FF symmetry breaking OPE

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s) + \mathcal{O}\left(\lambda^2\right), \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

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$C_7^{\rm SM} \approx -0.3,\ C_9^{\rm SM} \approx 4.2,\ C_{10}^{\rm SM} \approx -4.2$

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K^*}} V, \qquad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K^*})\,A_1, \qquad f_0 = \frac{(1-\hat{s}-\hat{M}_{K^*}^2)(1+\hat{M}_{K^*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K^*}(1+\hat{M}_{K^*})\sqrt{\hat{s}}}$$

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Subleading corrections to TransAmp's

$\lambda = \Lambda_{\mathrm{QCD}}/m_b \sim 0.15$

Low hadronic recoil

$\Rightarrow$ small, apart from possible duality violations

FF symmetry breaking OPE

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s) + \mathcal{O}\left(\lambda^2\right), \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

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$C_7^{\mathrm{SM}} \approx -0.3,\ C_9^{\mathrm{SM}} \approx 4.2,\ C_{10}^{\mathrm{SM}} \approx -4.2$

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1 + \hat{M}_{K^*}} V, \qquad f_\parallel = \sqrt{2}\,(1 + \hat{M}_{K^*})\, A_1, \qquad f_0 = \frac{(1 - \hat{s} - \hat{M}_{K^*}^2)(1 + \hat{M}_{K^*})^2 A_1 - \hat{\lambda}\, A_2}{2\,\hat{M}_{K^*}\,(1 + \hat{M}_{K^*})\sqrt{\hat{s}}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Large hadronic recoil

$$A_{\perp,\parallel}^{L,R} \sim \pm C_\perp^{L,R} \times \xi_\perp + \mathcal{O}(\alpha_s, \lambda), \qquad A_0^{L,R} \sim C_\parallel^{L,R} \times \xi_\parallel + \mathcal{O}(\alpha_s, \lambda)$$

2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

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Subleading corrections to TransAmp's

$\lambda = \Lambda_{\text{QCD}}/m_b \sim 0.15$

Low hadronic recoil

$\Rightarrow$ small, apart from possible duality violations

FF symmetry breaking — OPE

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s) + \mathcal{O}\left(\lambda^2\right), \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

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Large hadronic recoil

$\Rightarrow$ limited, end-point-divergences at $\mathcal{O}(\lambda)$

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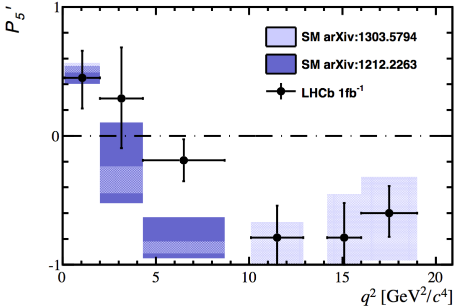
P_5' & subleading corrections

tension in P_5': 3.7σ for $q^2 \in [4.3, 8.7]$ GeV^2

2.5σ for $q^2 \in [1.0, 6.0]$ GeV^2

comparing experiment [LHCb arXiv:1308.1707]
with theory [Descotes-Genon/Hurth/Matias/Virto 1303.5794]

$\Rightarrow$ 2 “recipes” used to estimate subleading crr’s @ low q^2 (mainly for FF’s)



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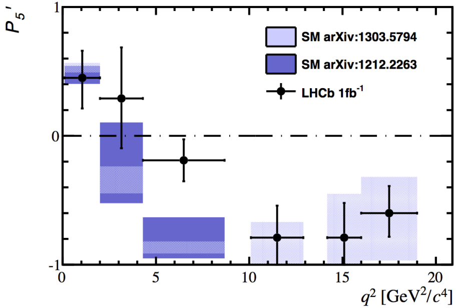

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⇒ 2 "recipes" used to estimate subleading crr's @ low q^2 (mainly for FF's)

I) Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589

Introduce "rescaling factor ζ" for each K^*-transversity amplitude

$$A^{L/R}_{0,\perp,\parallel} \longrightarrow \zeta^{L/R}_{0,\perp,\parallel} \times A_{0,\perp,\parallel} \qquad 1 - \frac{\Lambda_{\rm QCD}}{m_b} \lesssim \zeta \lesssim 1 + \frac{\Lambda_{\rm QCD}}{m_b}$$

- mimic subleading crr's from A) FF relations and B) $1/m_b$ contr. to ampl.
- can account for q^2-dep.: introduce ζ for each q^2-bin
- used in most analysis/fits

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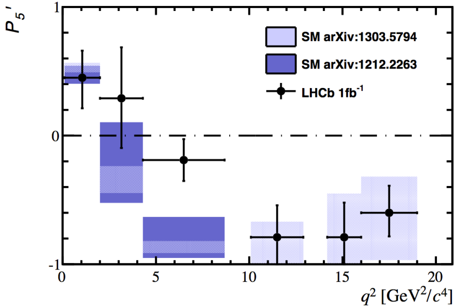


II) Jäger/Martin-Camalich arXiv:1212.2263 (updates in arXiv:1412.3183)

Keep track of subleadig crr.'s to FF-relations (ξ_j = universal FF)

$$FF_i \propto \xi_j + \alpha_s \Delta FF_i + a_i + b_i \frac{q^2}{m_B^2} + \ldots$$

with a_i, b_i from spread of nonperturbative FF-calculations (LCSR, quark models ...)

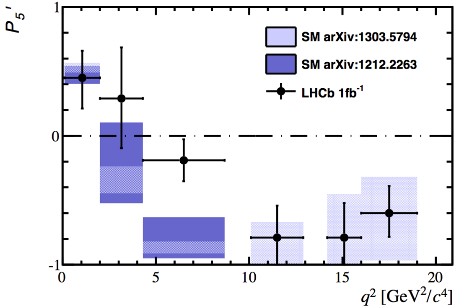
a_i, b_i are $\sim \Lambda_{\rm QCD}/m_b$ and ΔFF_i QCD crr's [Beneke/Feldmann hep-ph/0008255]

"Scheme-dependence" for definition of ξ_j in terms of QCD FF's

Scheme 1 $\quad \xi_\perp^{(1)} \equiv \frac{m_B}{m_B + m_{K^*}} V \qquad \xi_\parallel^{(1)} \equiv \frac{m_B + m_{K^*}}{2E} A_1 - \frac{m_B - m_{K^*}}{m_B} A_2$

Scheme 2 $\quad \xi_\perp^{(2)} \equiv T_1 \qquad \xi_\parallel^{(2)} \equiv \frac{m_{K^*}}{E} A_0$

P_5' & subleading corrections



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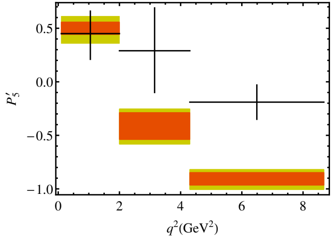
III) Descotes-Genon/Hofer/Matias/Virto arXiv:1407.8526

Update of method II) $\Rightarrow$ find smaller subleading FF corrections, contrary to II)

parametric + subleading $1/m_b$
$\bar{c}c$ estimate
LHCb data



- use LCSR results of FF's to estimate subleading $1/m_b$ contributions $\Rightarrow$ typically $\lesssim 10\%$
- contrary to II), do not fix central values of subleading contributions to zero, obtain them from fit
- contrary to II), use q^2-dep. of $\xi_{\perp,\parallel}$ as given by LCSR result of QCD FF's, do not use q^2-dep. as predicted by power count. in $m_b \to \infty$ limit
- Scheme 1 better for observables sensitive to $C_{9,10}$, Scheme 2 for observables $\sim C_7$

Angular analysis and "real life"

When aiming at precision measurements in $B \to K^*(\to K\pi)\bar{\ell}\ell$ (P-wave config)

- inclusion of resonant and non-resonant $K\pi$ (in S-wave config) important in experiments
  - $\Rightarrow$ additional contributions to angular distribution
  - $\Rightarrow$ $P-$ and $S-$wave can be disentangled in angular analysis
  - $\Rightarrow$ taken into account by LHCb and CMS

[Lu/Wang arXiv:1111.1513, Becirevic/Tayduganov 1207.4004, Blake/Egede/Shires 1210.5279, Matias 1209.1525]

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Extended angular analysis

- $B \to K\pi\bar{\ell}\ell$ off-resonance ($m^2_{K\pi} \neq m^2_{K^*}$) at high-$q^2$ [Das/Hiller/Jung/Shires arXiv:1406.6681]

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} \longrightarrow \frac{d^5\Gamma}{dm^2_{K\pi} dq^2 d\cos\theta_\ell d\cos\theta_K d\phi}$$

  - $\Rightarrow$ include contributions from $S-$, $P-$, and $D-$wave
  - $\Rightarrow$ provide access to further combinations of Wilson coefficients
  - $\Rightarrow$ probe strong phase differences with resonant contribution
  - $\Rightarrow$ analogously for $B_s \to \bar{K}K\bar{\ell}\ell$

- complementary constraints from angular analysis of $\Lambda_b \to \Lambda\bar{\ell}\ell$

[Böer/Feldmann/van Dyk arXiv:1410.2115]

Angular analysis of $B \to K \bar{\ell}\ell$

Besides $d\Gamma/dq^2$, two more obs's measured LHCb 3/fb arXiv:1403.8045

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{F_H}{2} + A_{\rm FB}\cos\theta_\ell + \frac{3}{4}\left[1 - F_H\right]\sin^2\theta_\ell$$

In the SM:

- $F_H \sim m_\ell^2/q^2$ tiny for $\ell = e, \mu$ and reduced FF uncertainties @ low- & high-q^2
  CB/Hiller/Piranishvili arXiv:0709.4174, CB/Hiller/van Dyk/Wacker arXiv:1111.2558
- $A_{\rm FB} \simeq 0 + \mathcal{O}(\alpha_e) + \mathcal{O}(\text{dim} - 8)$ up to "QED-background" & higher dim. m_b^2/m_W^2

Beyond SM: test scalar & tensor operators CB/Hiller/Piranishvili arXiv:0709.4174

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Lepton-flavour violating (LFV) effects: generalise $C_i \to C_i^\ell$!!!

Take ratios of observables for $\ell = \mu$ over $\ell = e$ (or $\ell = \tau$) Krüger/Hiller hep-ph/0310219

$\Rightarrow$ FF's cancel in SM up to $\mathcal{O}(m_\ell^4/q^4)$ @ low-q^2 CB/Hiller/Piranishvili arXiv:0709.4174

$$R_M^{[q^2_{\rm min},\, q^2_{\rm max}]} = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \frac{d\Gamma[B \to M\,\bar{\mu}\mu]}{dq^2}}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \frac{d\Gamma[B \to M\,\bar{e}e]}{dq^2}}$$

for $M = K, K^*, X_s$

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for $M = K, K^*, X_s$

Recent measurement of

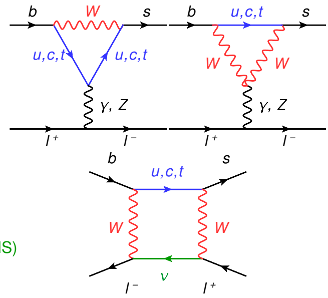
$R_K^{[1,6]} = 0.745^{+0.090}_{-0.074} \pm 0.036$ LHCb 3/fb arXiv:1406.6482

deviates by 2.6σ from SM

$R_{K,\rm SM}^{[1,6]} = 1.0008 \pm 0.0004$ Bouchard et al. arxiv:1303.0434

$B_s \to \bar{\mu}\mu$ at higher order in the Standard Model - I

Motivation



Th: test of the SM at loop-level (FCNC decay)
$\Rightarrow$ only hadronic uncertainty from $B_{d,s}$ decay constant
additional helicity suppression
$\Rightarrow$ sensitivity to beyond-SM (pseudo-) scalar interactions

Exp: important *B*-decay @ LHCb, CMS & ATLAS

$$\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)_{\rm Exp} = (2.8^{+0.7}_{-0.6}) \times 10^{-9} \qquad (6.2\sigma)$$

$$\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu)_{\rm Exp} = (3.9^{+1.6}_{-1.4}) \times 10^{-10} \qquad (3.2\sigma)$$

from 2013
(LHCb + CMS)

$\Rightarrow$ exp. prospects: ~ 5 % error with 50 fb^{-1} @ LHCb

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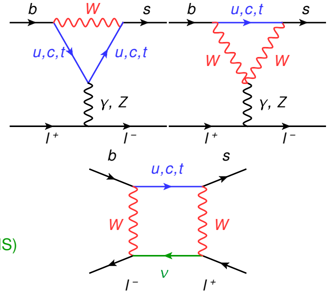

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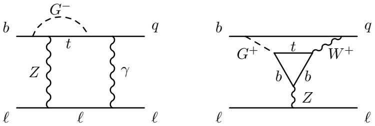
NLO electroweak (EW) corrections

[CB/Gorbahn/Stamou arXiv:1311.1348]

!!! LO EW theory unc.: $\gtrsim 7\%$ [Buras *et al.* arXiv:1208.0934]
(from different EW renormalization schemes)

- NLO EW matching ($\mu_0 \sim 160$ GeV) in 3 different schemes $\Rightarrow$ convergence: $0.3\% \lesssim$ deviation
- size of NLO correction: $\sim (3\ldots5)\%$ (dep on μ_0)
- resummation of NLO QED logarithms from $\mu_0 \to \mu_b \sim 5$ GeV: residual μ_b-dep. $\lesssim 0.3$ %



reduced EW uncertainty

@ LO:		$\gtrsim$	7%
@ NLO:	0.6%	$\lesssim$	

$B_s \to \bar{\mu}\mu$ at higher order in the Standard Model - II

- NNLO QCD crrs. reduce μ_0-dep. from 1.8% at NLO → 0.2% at NNLO
  [Hermann/Misiak/Steinhauser arXiv:1311.1347]

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Standard Model predictions @ (NLO EW + NNLO QCD)

$$\overline{\mathcal{B}}(B_s \to \bar{\mu}\mu)_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9}$$

$$\overline{\mathcal{B}}(B_d \to \bar{\mu}\mu)_{\rm SM} = (1.06 \pm 0.09) \times 10^{-10}$$

[CB/Gorbahn/Hermann/Misiak/Stamou/Steinhauser arXiv:1311.0903]

Error budget

	f_{B_q}	CKM	τ_H^q	M_t	α_s	other param.	non-param.	Σ
$\overline{\mathcal{B}}_{s\mu}$	4.0%	4.3%	1.3%	1.6%	0.1%	< 0.1%	1.5%	6.4%
$\overline{\mathcal{B}}_{d\mu}$	4.5%	6.9%	0.5%	1.6%	0.1%	< 0.1%	1.5%	8.5%

Non-parametric uncertainties:

- 0.3% from $\mathcal{O}(\alpha_{em})$ corrections from $\mu_b \in [m_b/2, 2m_b]$
- $2 \times 0.2\%$ from $\mathcal{O}(\alpha_s^3, \alpha_{em}^2, \alpha_s\alpha_{em})$ matching corrections from $\mu_0 \in [m_t/2, 2m_t]$
- 0.3% from top-mass conversion from on-shell to $\overline{\rm MS}$ scheme
- 0.5% further uncertainties (power corrections $\mathcal{O}(m_b^2/M_W^2), \ldots$)