



## Systematic approach to non-local charm contributions in exclusive $b \rightarrow sll$ decays

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based on Bobeth/Chrzaszcz/DvD/Virto 1705.XXXXX

SM@LHC in Amsterdam  
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## Effective theory for $b \rightarrow s$ transitions

For  $\Lambda_{\text{EW}}, \Lambda_{\text{NP}} \gg M_B$  : General model-independent parametrization of NP :

$$\mathcal{L}_W = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i(\mu) \mathcal{O}_i(\mu)$$

$$\mathcal{O}_1 = (\bar{c}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a c) \quad \mathcal{O}_2 = (\bar{c}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L c)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu} \quad \mathcal{O}_{7'} = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}$$

$$\mathcal{O}_{9\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) \quad \mathcal{O}_{9'\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_{10\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell) \quad \mathcal{O}_{10'\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),$$

SM contributions to  $\mathcal{C}_i(\mu_b)$  known to NNLL [ Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04; Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06]

$$\mathcal{C}_{7\text{eff}}^{\text{SM}} = -0.3, \mathcal{C}_{9\ell}^{\text{SM}} = 4.1, \mathcal{C}_{10\ell}^{\text{SM}} = -4.3, \mathcal{C}_1^{\text{SM}} = -0.4, \mathcal{C}_2^{\text{SM}} = 1.1, \mathcal{C}_{\text{rest}}^{\text{SM}} \lesssim 10^{-2}$$



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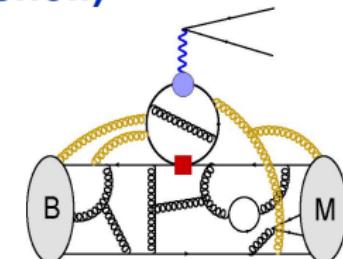
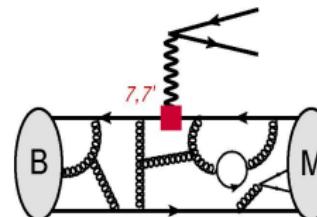
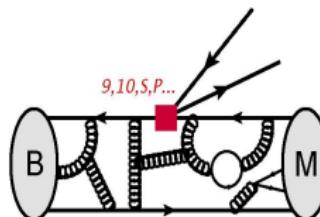
$$\mathcal{O}_{10'\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),$$

numerically leading operators

source of largest systematic uncertainty

$$\mathcal{C}_{7\text{eff}}^{\text{SM}} = -0.3, \mathcal{C}_{9\ell}^{\text{SM}} = 4.1, \mathcal{C}_{10\ell}^{\text{SM}} = -4.3, \mathcal{C}_1^{\text{SM}} = -0.4, \mathcal{C}_2^{\text{SM}} = 1.1, \mathcal{C}_{\text{rest}}^{\text{SM}} \lesssim 10^{-2}$$

## Theory of exclusive $B \rightarrow M\ell^+\ell^-$ (in a nutshell)



$$\mathcal{M}_\lambda = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[ (\mathcal{A}_\lambda^\mu + \mathcal{H}_\lambda^\mu) \bar{u}_\ell \gamma_\mu v_\ell + \mathcal{B}_\lambda^\mu \bar{u}_\ell \gamma_\mu \gamma_5 v_\ell \right] + \mathcal{O}(\alpha^2)$$

**Local:**  $\mathcal{A}_\lambda^\mu = -\frac{2m_b q_\nu}{q^2} \mathcal{C}_7 \langle M_\lambda | \bar{s} \sigma^{\mu\nu} P_R b | B \rangle + \mathcal{C}_9 \langle M_\lambda | \bar{s} \gamma^\mu P_L b | B \rangle$

$$\mathcal{B}_\lambda^\mu = \mathcal{C}_{10} \langle M_\lambda | \bar{s} \gamma^\mu P_L b | B \rangle$$

**Non-Local:**  $\mathcal{H}_\lambda^\mu = -\frac{16i\pi^2}{q^2} \sum_{i=1..6,8} \mathcal{C}_i \int d^4x e^{iq \cdot x} \langle M_\lambda | T \{ \mathcal{J}_{\text{em}}^\mu(x), \mathcal{O}_i(0) \} | B \rangle$

### Two theory issues:

1. **form factors**

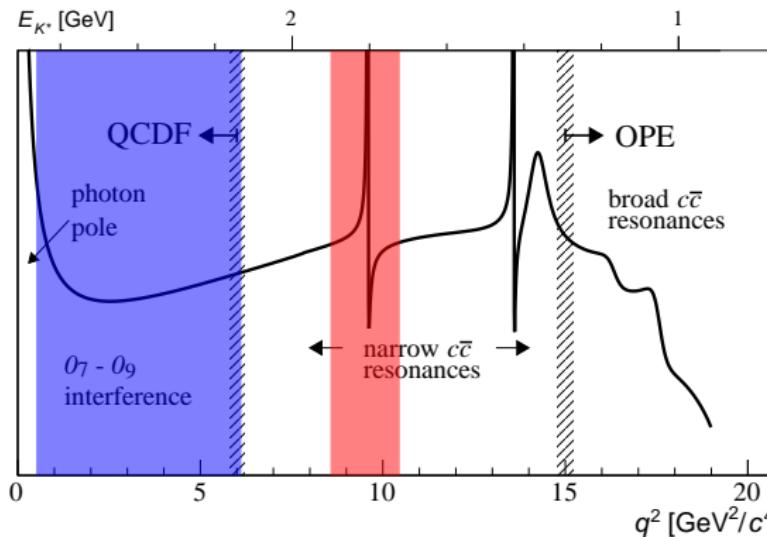
(LCSR, LQCD, symmetry relations ...)

2. **nonlocal hadronic contribution**

(SCET/QCDF, OPE, LCOPE ...)

## The decay $B \rightarrow K^* \mu^+ \mu^-$

[sketch taken from Blake/Gershon/Hiller 1501.03309]

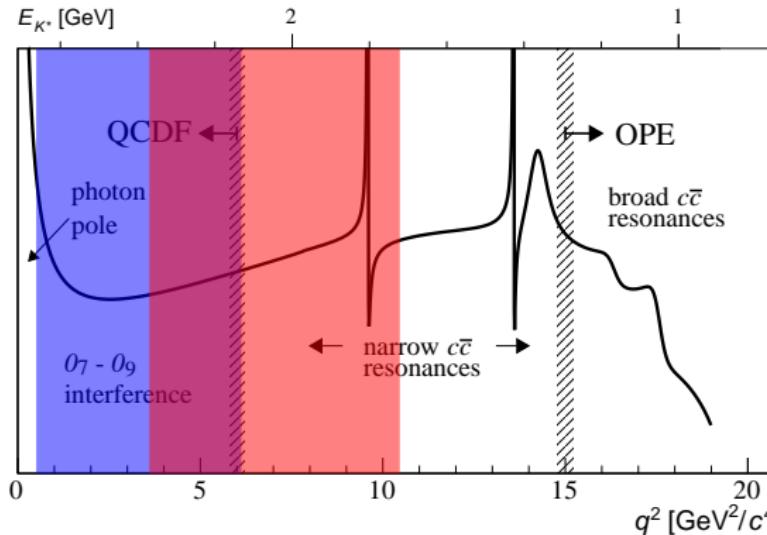


region of interest (for this talk)

polluted by  $B \rightarrow K^* J/\psi$  "background"

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[sketch taken from Blake/Gershon/Hiller 1501.03309]



region of interest (for this talk)

polluted by  $B \rightarrow K^* J/\psi$  "background"

- uncertainty w.r.t. onset of  $B \rightarrow K^* J/\psi$  pollution poses largest systematic theory uncertainty



## Hadronic correlator : Decomposition

$$\begin{aligned}\mathcal{H}^\mu(q^2) &\equiv i \int d^4x e^{iq \cdot x} \langle \bar{K}^*(k, \eta) | T \{ \bar{c} \gamma^\mu c(x), \mathcal{C}_1 \mathcal{O}_1 + \mathcal{C}_2 \mathcal{O}_2(0) \} | \bar{B}(p) \rangle \\ &\equiv M_B^2 \eta_\alpha^* \left[ S_\perp^{\alpha\mu} \mathcal{H}_\perp - S_{||}^{\alpha\mu} \mathcal{H}_{||} - S_0^{\alpha\mu} \mathcal{H}_0 \right]\end{aligned}$$

here

$S_\lambda^{\alpha\mu}$  basis of Lorentz structures (carefully chosen)

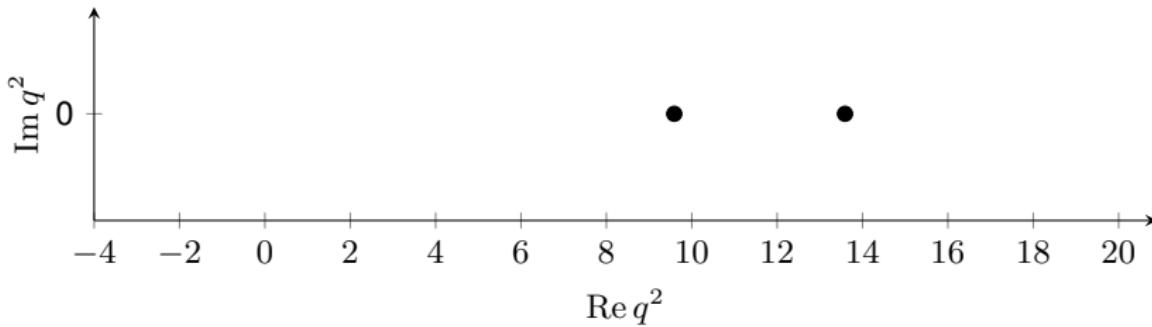
$\mathcal{H}_\lambda$  Lorentz invariant correlation functions

$\lambda$  polarization states ( $\perp, ||, 0$ )

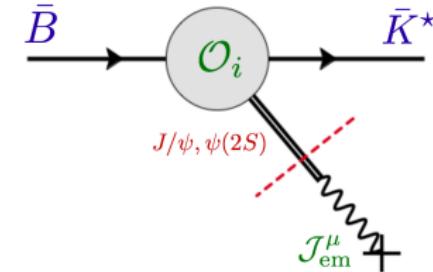
our idea:

- understand analytic structure of the  $\mathcal{H}_\lambda(q^2)$  to write a general parametrisation consistent with QCD.

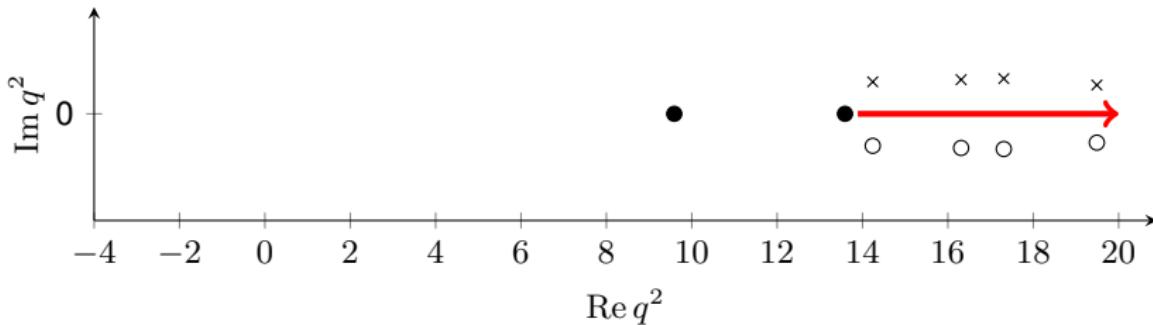
## Hadronic correlator : Analytic structure



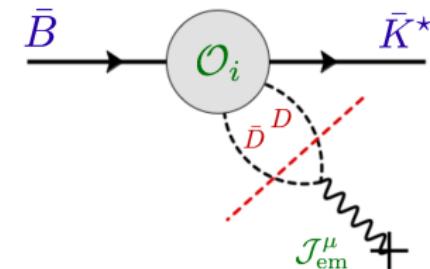
- narrow charmonia, assumed to be stable



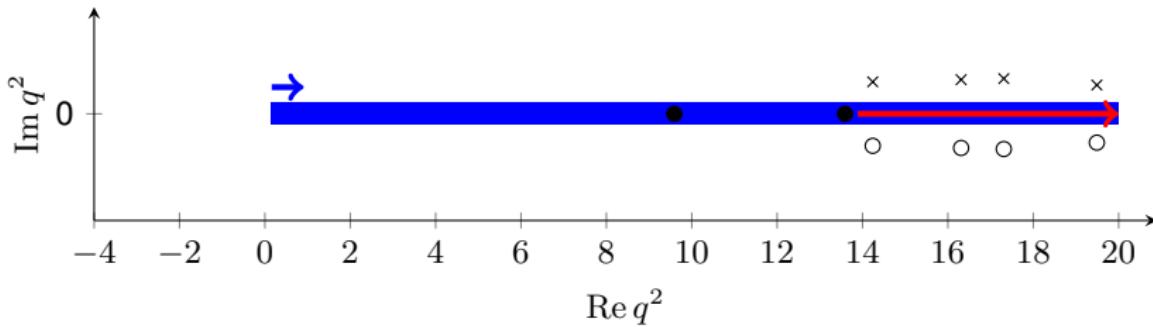
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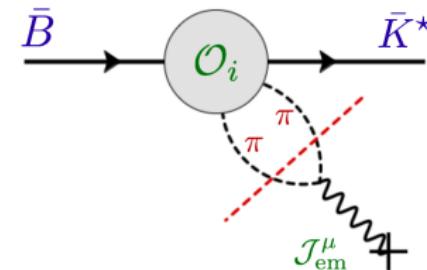
- narrow charmonia, assumed to be stable
- red branch cut from  $D\bar{D}$  production
- broad charmonia, decaying to  $D\bar{D}$
- × potential mirror poles



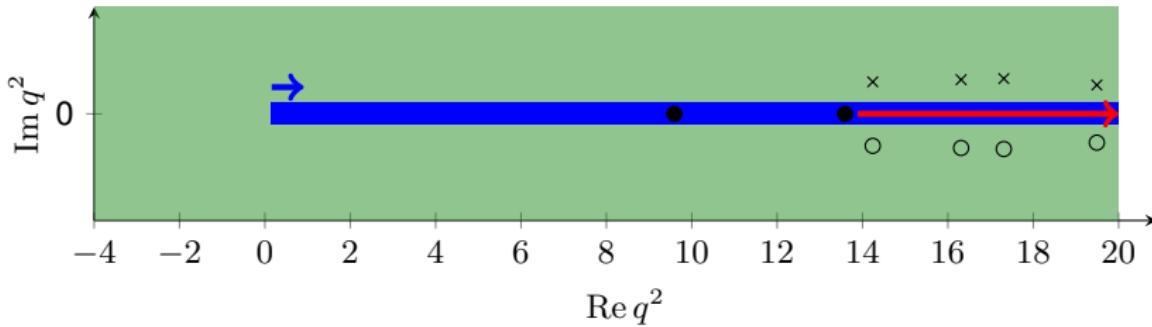
## Hadronic correlator : Analytic structure



- narrow charmonia, assumed to be stable
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## Hadronic correlator : Analytic structure



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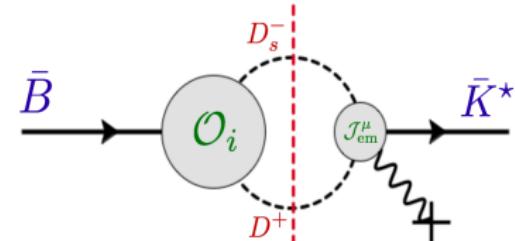
red branch cut from  $D\bar{D}$  production

○ broad charmonia, decaying to  $D\bar{D}$

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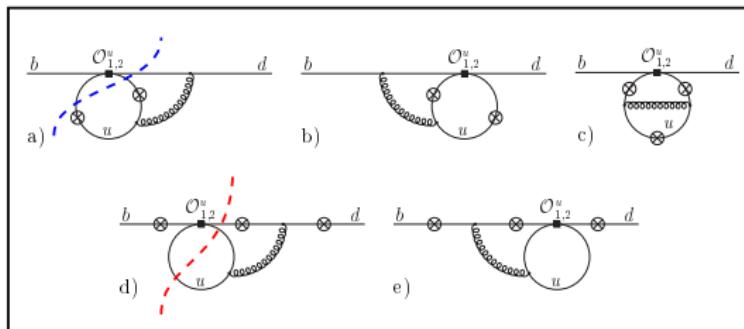
blue branch cut from light hadrons

green  $q^2$ -dep. imaginary part  
(due to branch cut in  $p^2$ )



## Understanding the $p^2$ cut

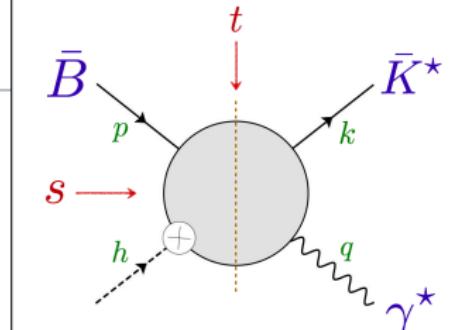
- $p^2$  cut arises from loop contributions
  - at one loop: only right-hand cut in  $q^2$
  - at two loop: no analytic results available for  $b \rightarrow s\bar{c}\bar{c}$   
series expansion available
- [see e.g. Greub/Pilipp/Schupbach 2008]
- turn to  $b \rightarrow d\bar{u}\bar{u}$  instead as a toy analysis, for which analytic two-loop results are available
- [Seidel 2004]



- at two loop: two sources of  $p^2$  cut
  - $u\bar{u}d$  on shell at  $q^2 < 0$
  - $\bar{d}d\bar{d}$  on shell at  $q^2 < 0$

## Understanding the $p^2$ cut

**Trick:** Add spurious momentum  $h$  to  $\mathcal{O}_i$   
 Recover physical kinematics as  $h \rightarrow 0$



- consider Mandelstam variables

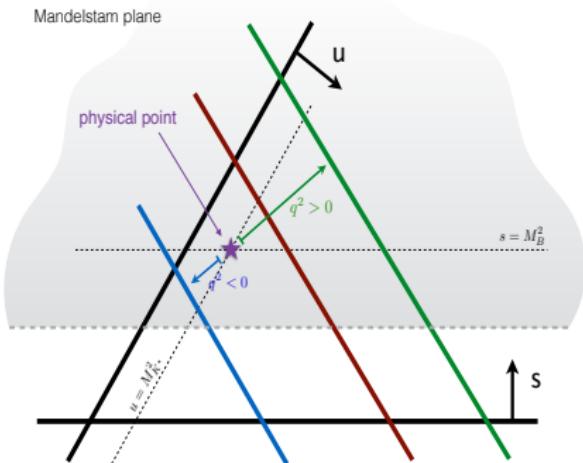
$$s \equiv (p + h)^2 \quad \longrightarrow \quad M_B^2$$

$$u \equiv (k - h)^2 \quad \longrightarrow \quad M_{K^*}^2$$

$$t \equiv (q - h)^2 \quad \xrightarrow{\text{physical point}} \quad q^2$$

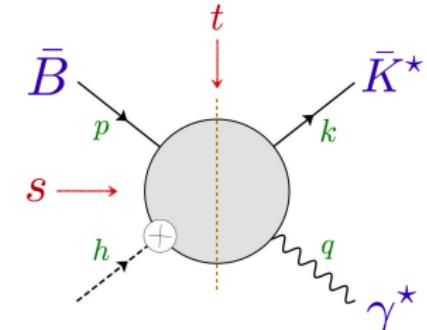
- $s$  independent of  $t$

- cut in  $s \sim p^2$  does not translate into cut in  $t \sim q^2$



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- two correlators:

$$\mathcal{H}_\lambda(q^2) \rightarrow \mathcal{H}_\lambda^{\text{real}}(q^2) + i \mathcal{H}_\lambda^{\text{imag}}(q^2)$$

- both are analytic at  $q^2 \leq 0$
- both have branch cuts at  $q^2 > 0$
- the same dispersion relation governs their  $q^2$ -dependence

- $s$  independent of  $t$

- cut in  $s \sim p^2$  does not translate into cut in  $t \sim q^2$



## Setup

**task 1** parametrize the correlator  $\mathcal{H}_\mu$

- take care to preserve the analytic structure

**task 2** fit the parametrization to *suitable* inputs

- a theoretical inputs far away from the onset of the  $J/\psi$  pole
- b experimental input on the  $J/\psi$  and  $\psi(2S)$  pole

**task 3** predict observables in  $B \rightarrow K^* \mu^+ \mu^-$  decays

- a assume the Standard Model holds
- b assume a benchmark point  $\mathcal{C}_9^{\text{NP}} = -1$ , compatible with present global fits

**task 4** fit the parametrization also to  $B \rightarrow K^* \mu^+ \mu^-$  data

- a test if the predictions fit the data
- b test whether modifications to  $\mathcal{C}_9^{\text{NP}}$  improve the fit

## Parametrization A : $J/\psi, \psi(2s)$ poles + $D\bar{D}$ cut    (task 1 ✓)

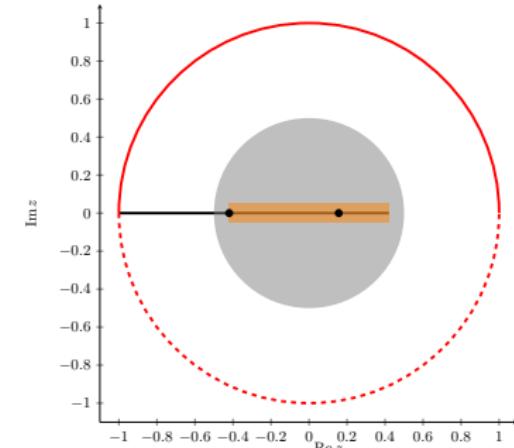
Motivated by famous “ $z$ -parametrization” of form factors

[Boyd/Grinstein/Lebed hep-ph/9412324]

1. extract the poles

$$\mathcal{H}_\lambda(q^2) \sim \frac{1}{q^2 - M_{J/\psi}^2} \frac{1}{q^2 - M_{\psi(2S)}^2} \hat{\mathcal{H}}_\lambda(q^2)$$

2.  $\hat{\mathcal{H}}_\lambda(q^2)$  is analytic except for  $D\bar{D}$  cut.
3. Perform conformal mapping  $q^2 \mapsto z(q^2)$
4.  $\hat{\mathcal{H}}_\lambda(z)$  analytic within unit circle  $|z| = 1$
5. Taylor expand  $\hat{\mathcal{H}}_\lambda(z)$  around  $z = 0$
6. Good convergence expected since  $|z| < 0.48$  for  $-5 \text{ GeV}^2 \leq q^2 \leq 14 \text{ GeV}^2$





## Theory inputs

(task 2a ✓)

the correlator can be calculated reliably at  $q^2 < 0$  by means of a light-cone OPE

[Khodjamirian et al. 1006.4945]

using  $\mathcal{H}_\perp(q^2)$  as an example:

$$\begin{aligned}\mathcal{H}_\perp(q^2) = & \left[ \# \times g(q^2, m_c^2) + \frac{\alpha_s}{4\pi} \times \# \times \{F_1^9, \dots\} \right] \times \mathcal{F}_\perp(q^2) \\ & + \# \times \tilde{\mathcal{V}}_1(q^2) \\ & + \# \times \phi_B \otimes \mathcal{T}_\perp \otimes \phi_{K^*}\end{aligned}$$

- first line contains form-factor-like contributions
  - LO contribution
  - NLO correction (produces  $p^2$  cut !!) [Asatryan et al. hep-ph/0109140]
- third term arises from soft-gluon effects only [Khodjamirian et al. 1006.4945]
- fourth term arises from hard-gluon effects only (w/ spectator)

[Beneke et al. hep-ph/0106067 and hep-ph/0412400]

details:

- compute  $\mathcal{H}_\lambda/\mathcal{F}_\lambda$  at  $q^2 = -1 \text{ GeV}^2$  and  $q^2 = -5 \text{ GeV}^2$
- include (substantial) correlations across correlators and across  $q^2$



## Experimental inputs

(task 2b ✓)

correlators  $\mathcal{H}_\lambda$  can be related to observables in the decays  $B \rightarrow K^* \{J/\psi, \psi(2S)\}$

- independent of short-distance contributions ( $\mathcal{C}_7, \mathcal{C}_9$ , etc) in  $B \rightarrow K^* \{\gamma, \mu^+ \mu^-\}$
- important constraints at  $q^2 \simeq 9 \text{ GeV}^2$  and  $q^2 \simeq 14 \text{ GeV}^2$

details:

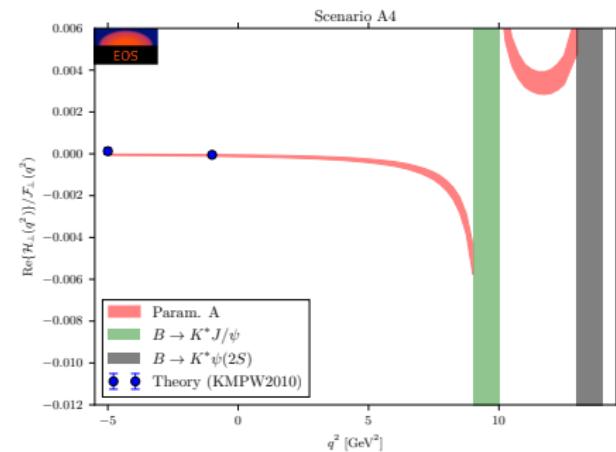
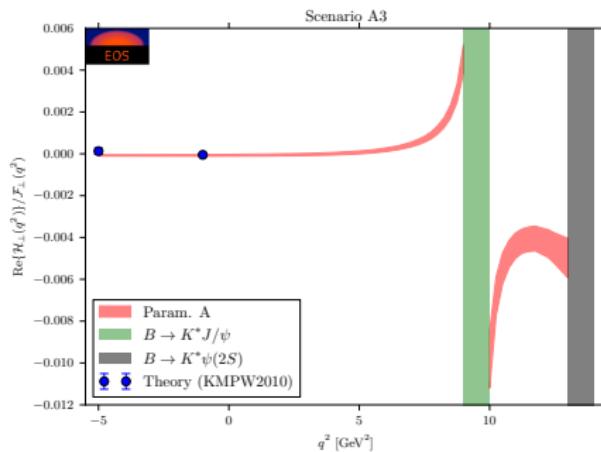
- residues of the correlator can be expressed in terms of  $B \rightarrow K^* \psi$  amplitudes
- $\mathcal{B}$  and 4 angular observables measured in  $B \rightarrow K^* J/\psi$  and  $B \rightarrow K^* \psi(2S)$

[BaBar 2007, Belle 2014, LHCb 2013]

- allows to constrain all moduli and two relative phases of the amplitudes, and therefore of the residues of the correlator.

## Preliminary! Predictions for the correlator

Results for  $\text{Re}(\mathcal{H}_\perp/\mathcal{F}_\perp)$ :



discrete ambiguity in phases of the residues: (only two shown)

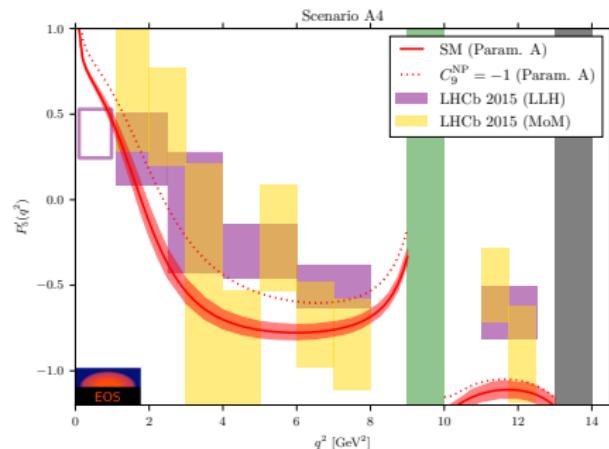
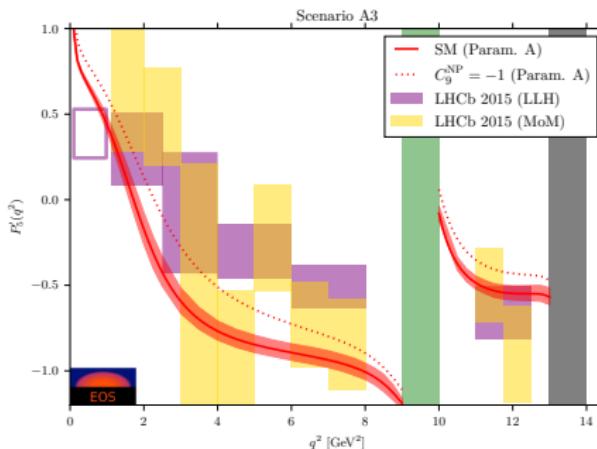
**Left:**  $\phi_{J/\psi} = \pi$ ,  $\phi_{\psi(2S)} = 0$

**Right:**  $\phi_{J/\psi} = \phi_{\psi(2S)} = \pi$

## Preliminary! Predictions for $P'_5$

(tasks 3a,3b ✓)

prediction within Standard Model (SM) and one benchmark point ( $C_9^{\text{NP}} = -1$ )



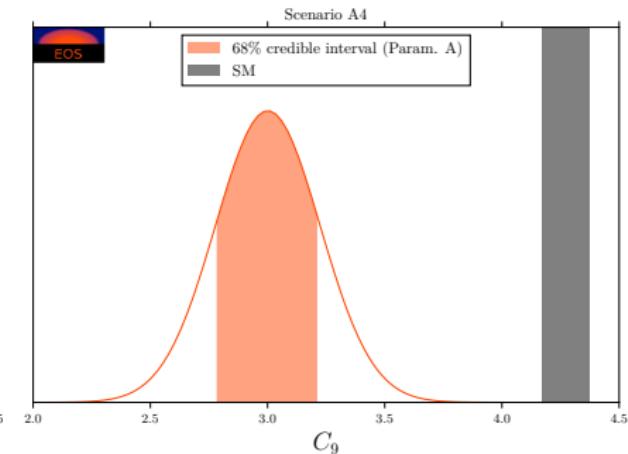
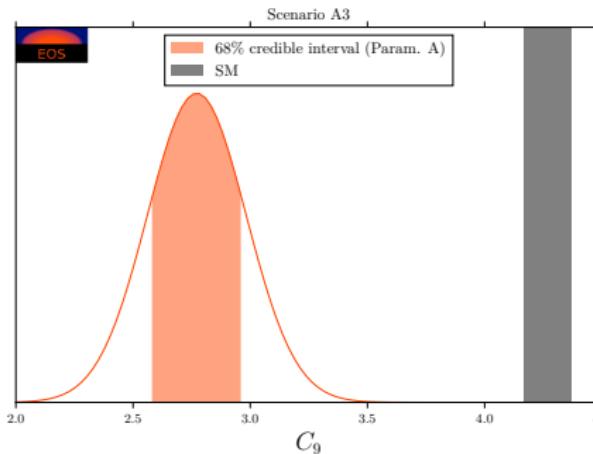
**Left :**  $\phi_{J/\psi} = \pi$ ,  $\phi_{\psi(2S)} = 0$

**Right :**  $\phi_{J/\psi} = \phi_{\psi(2S)} = \pi$

- **great potential:** for the first-time, fits gain sensitivity to inter-resonance bin
  - more comprehensive usage of available data
  - complementary information compared to below-resonance data

## Preliminary! Challenge $B \rightarrow K^* \mu^+ \mu^-$ data (task 4b ✓)

Global fit to all  $B \rightarrow K^* \{ \gamma, \mu^+ \mu^-, J/\psi, \psi(2S) \}$  data using Parametrization A



**Left :**  $\phi_{J/\psi} = \pi$ ,  $\phi_{\psi(2S)} = 0$

**Right :**  $\phi_{J/\psi} = \phi_{\psi(2S)} = \pi$

- for all choice of phases: pull >  $4\sigma$



## Summary and outlook

- framework to access nonlocal correlator
  - first approach to use both theory inputs and experimental constraints in a fit
  - can accommodate existing and future theory results (systematically improvable)
  - provides model-independent prior predictions for  $B \rightarrow K^{(*)}\mu^+\mu^-$
  - can be easily embedded in global fits
- present data in tension with parametrization A
  - favours NP interpretation with  $> 4\sigma$
- other results not shown here
  - complex parametrization A: needs NLO terms
  - parametrization B: includes light-hadron cut from  $\psi$  decay

[Bobeth/Chrzaszcz/DvD/Virto 1705.xxxxx]

[analytic  $\rightarrow$  Greub/Virto w.i.p.]



## Wish list for our experimental colleagues

- rate and full set of angular observables of  $B \rightarrow K^* \psi(\rightarrow \mu^+ \mu^-)$  for both narrow charmonium resonances
  - $\psi(2S)$  not available from LHCb!
  - provide results *including* the  $Z_c$  states removal of  $Z_c$  states in recent analyses (e.g. by Belle) complicates our analysis
- smaller charmonium veto windows
- three or more bins between  $J/\psi$  and  $\psi(2S)$  for rate and angular observables
  - partial overlap with previous veto windows encouraged!