Improved Theory Predictions in $b \to s\ell\ell$

Quirks in Quark Flavor Physics – 14/06/2022

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In collaboration with:
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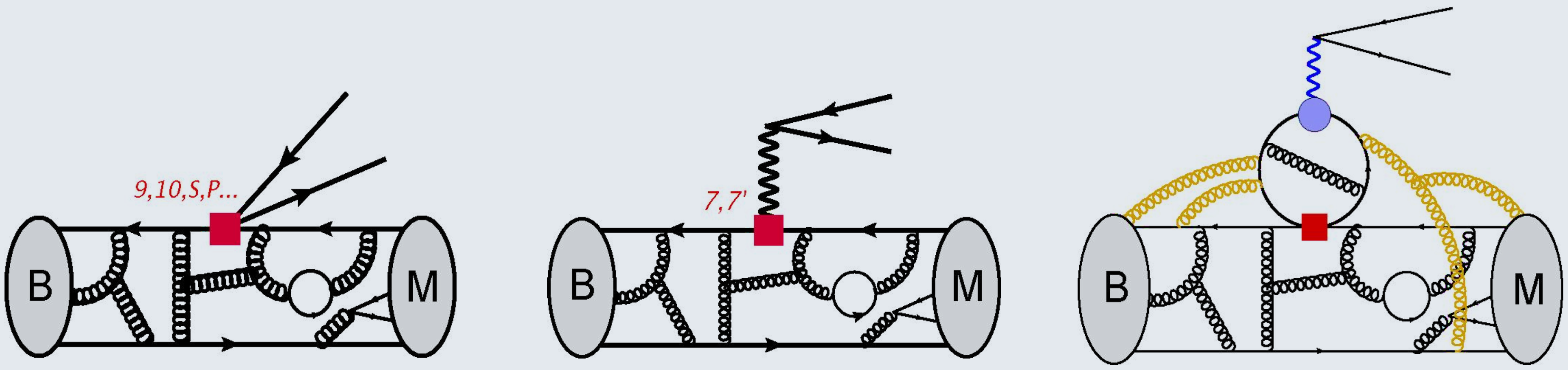
Form-factors in $b \to s\ell\ell$

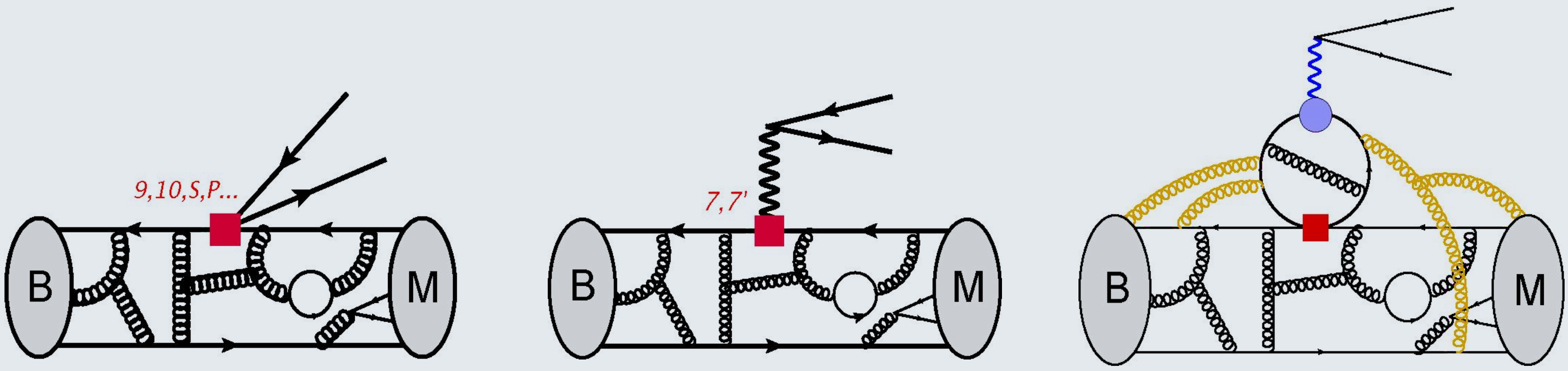


$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- $B \to K\,\mu\mu$, $B \to K^*\,\mu\mu$
- $B_s \to \varphi\,\mu\mu$
- $\Lambda_b \to \Lambda\,\mu\mu$, ...

We focus on these **3 channels** (and their isospin partners)!

Form-factors in b → sℓℓ



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Local form-factors

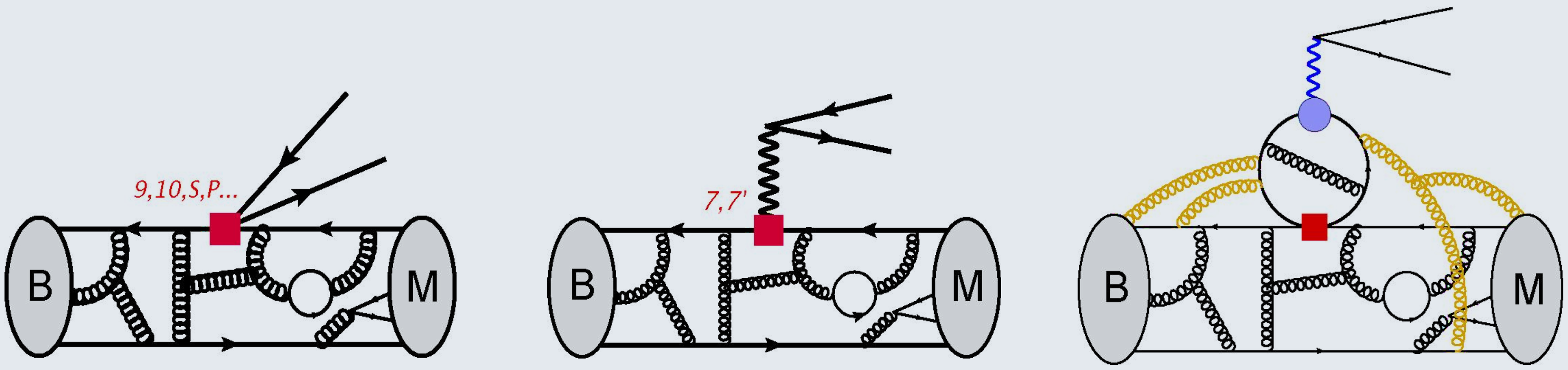
$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k) | \bar{s}\gamma_\mu b_L | \bar{B}(q+k) \rangle$$

Updated predictions based on:

- **Lattice QCD** calculations
- **Light-cone sum rules** estimates

... more in backup.

Form-factors in $b \to s\ell\ell$



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Non-local form-factors

$$\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{ \mathcal{J}_{\text{em}}^\mu(x), \mathcal{C}_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$$

→ Main contributions: $\mathcal{O}_1^c, \mathcal{O}_2^c$ the so-called **“charm-loops”**

Estimation of H_λ

QCD factorization in heavy quark limit [Beneke, Feldmann, Seidel, '01 & '04]

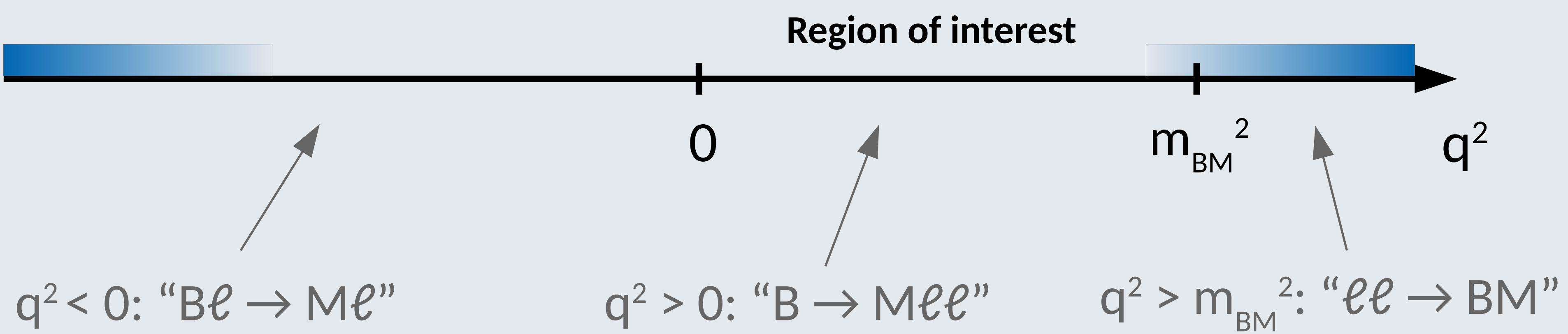
- Uses relations between the **local** form-factors
- + **perturbative** contributions from the charm loops

→ **Limited control** on the uncertainties

→ **Knows nothing about the J/ψ and the ψ(2S)!**

Constraints on H_λ

1. Two types of **OPE** can be used for H_λ:
   - **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol 2004][Beylich, Buchalla, Feldmann 2011]
     → We will discuss it later



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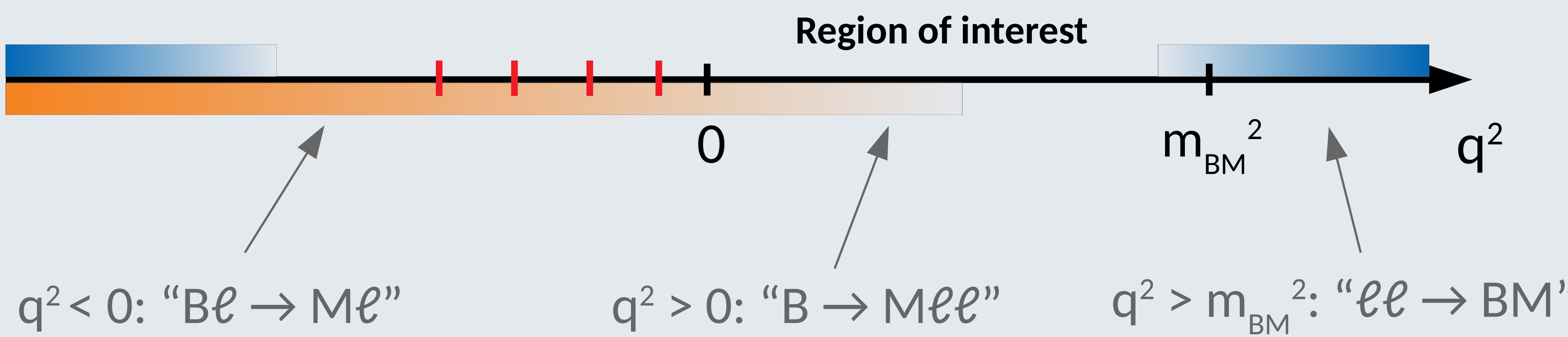
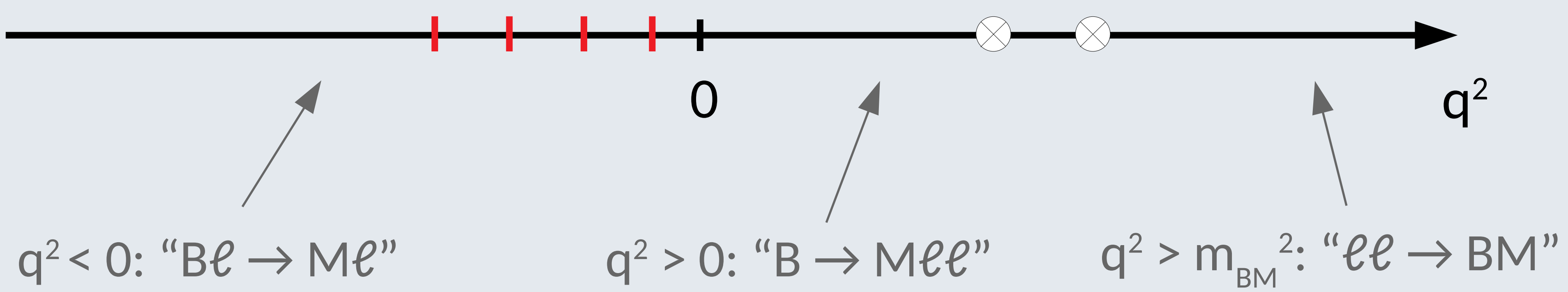
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang 2010]

  → theory points at $q^2 < 0$ [Gubernari, van Dyk, Virto 2020]



Constraints on H_λ

2. Charmonium resonances [Bobeth, Chrzaszsz, van Dyk, Virto'17]:
   - H_λ presents **poles** at $q^2 = m_{J/\psi}^2$ and $m_{\psi(2S)}^2$
   - For this work we only use **B → M J/ψ** data



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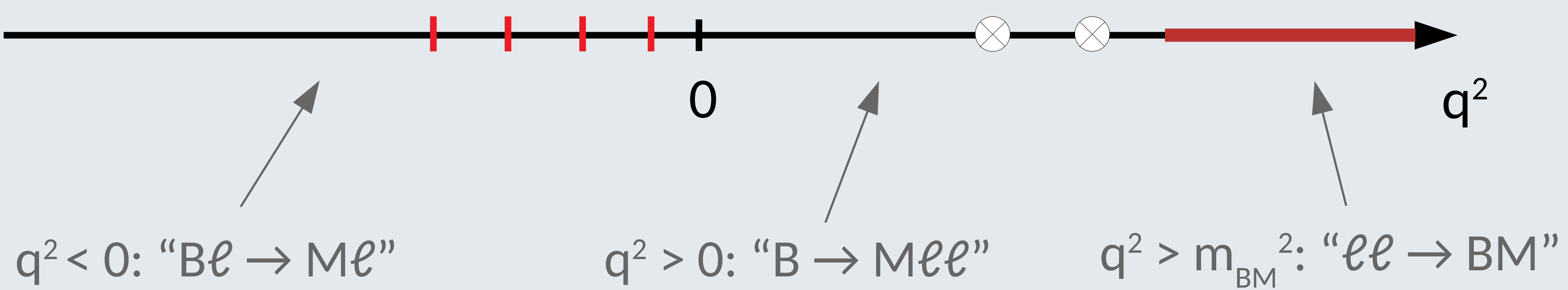
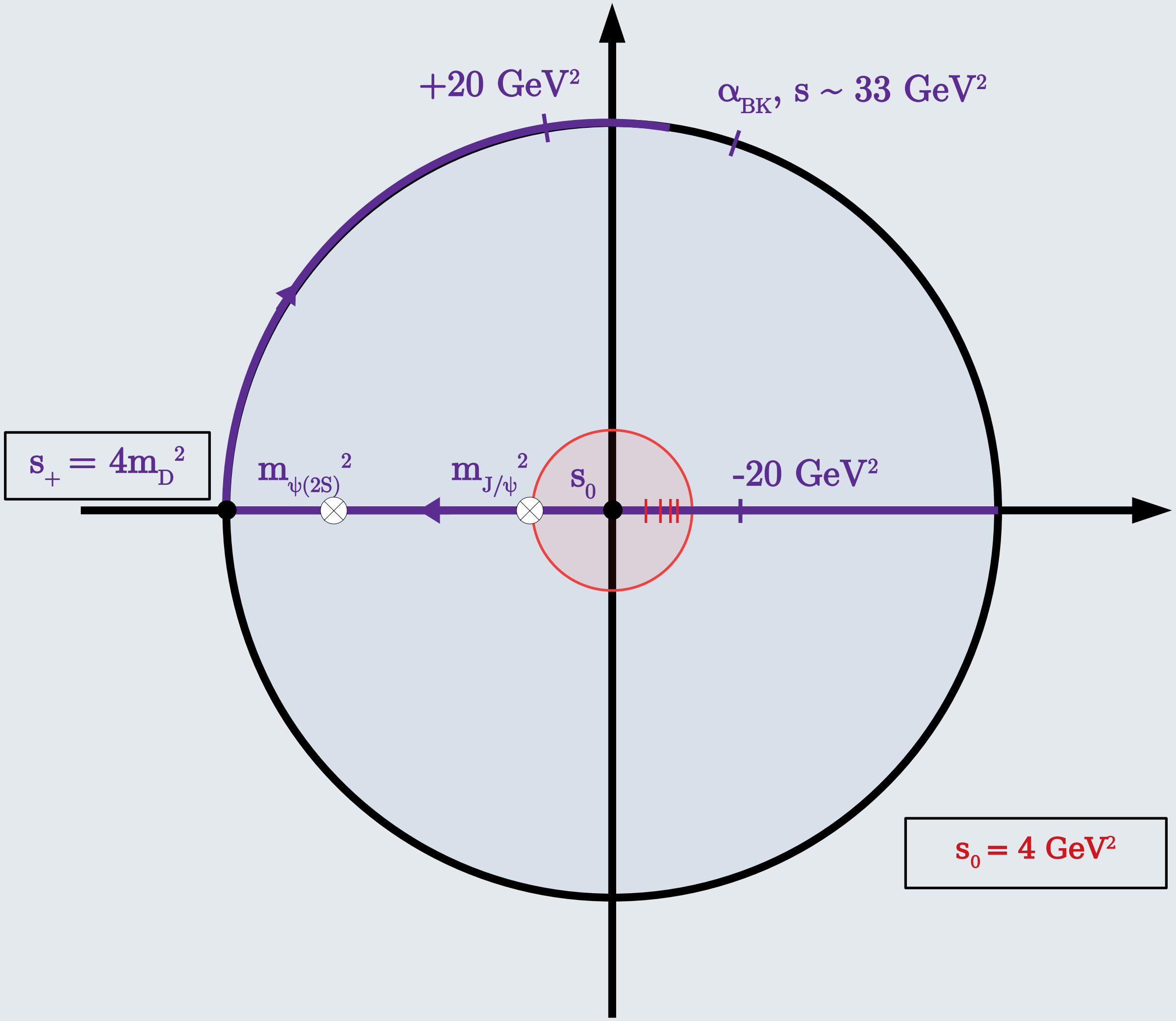
3. H_λ has a **branch cut** for $q^2 > 4m_D^2$



Parametrization of H_λ

- **z-mapping**

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$



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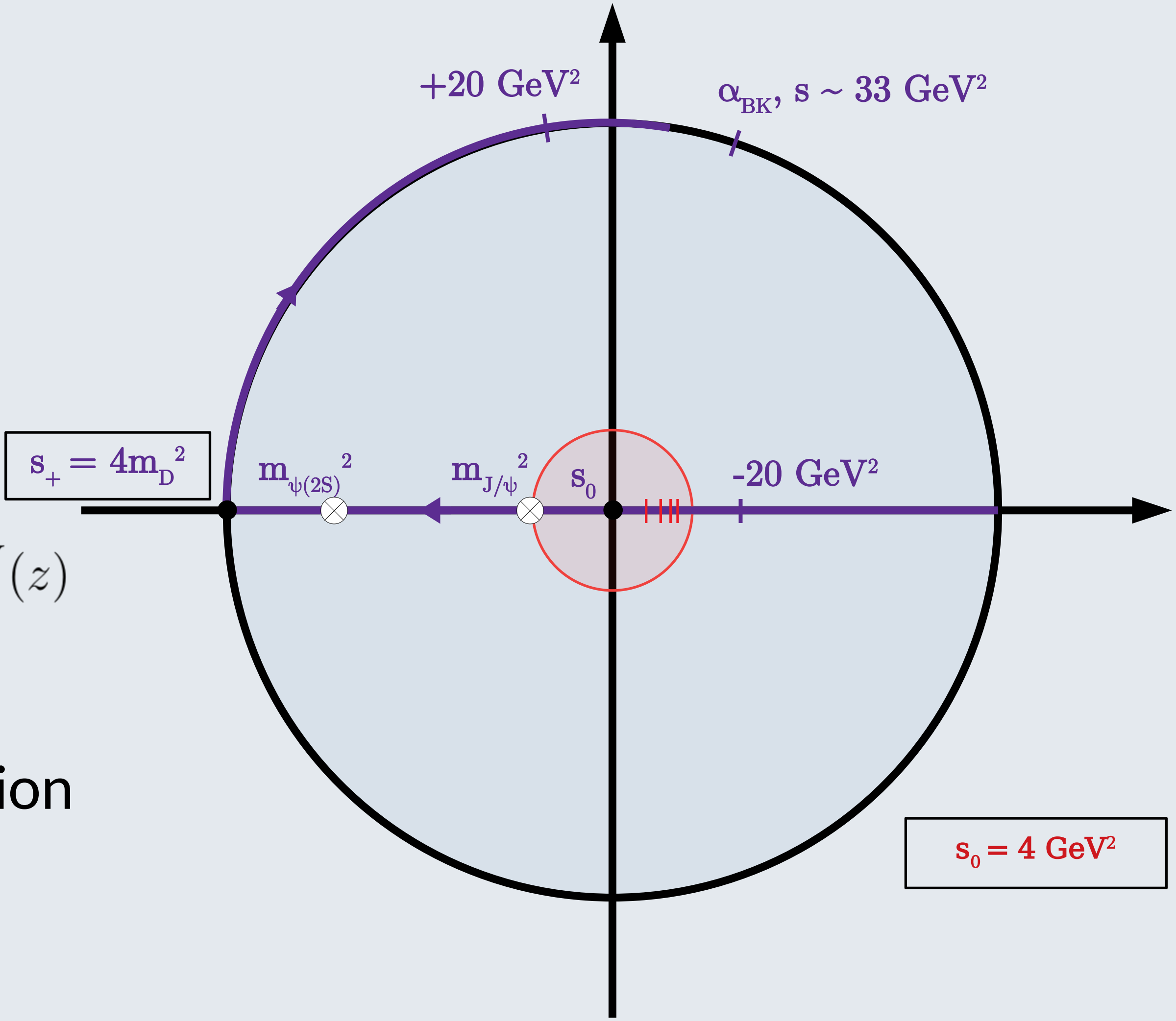
- **Analyticity**

$$\hat{\mathcal{H}}_\lambda^{B\to V}(z) \equiv \phi_\lambda^{B\to V}(z)\,\mathcal{P}(z)\,\mathcal{H}_\lambda^{B\to V}(z)$$

→ $\mathcal{P}(z)$ captures the poles

→ $\Phi(z)$ is a useful normalization



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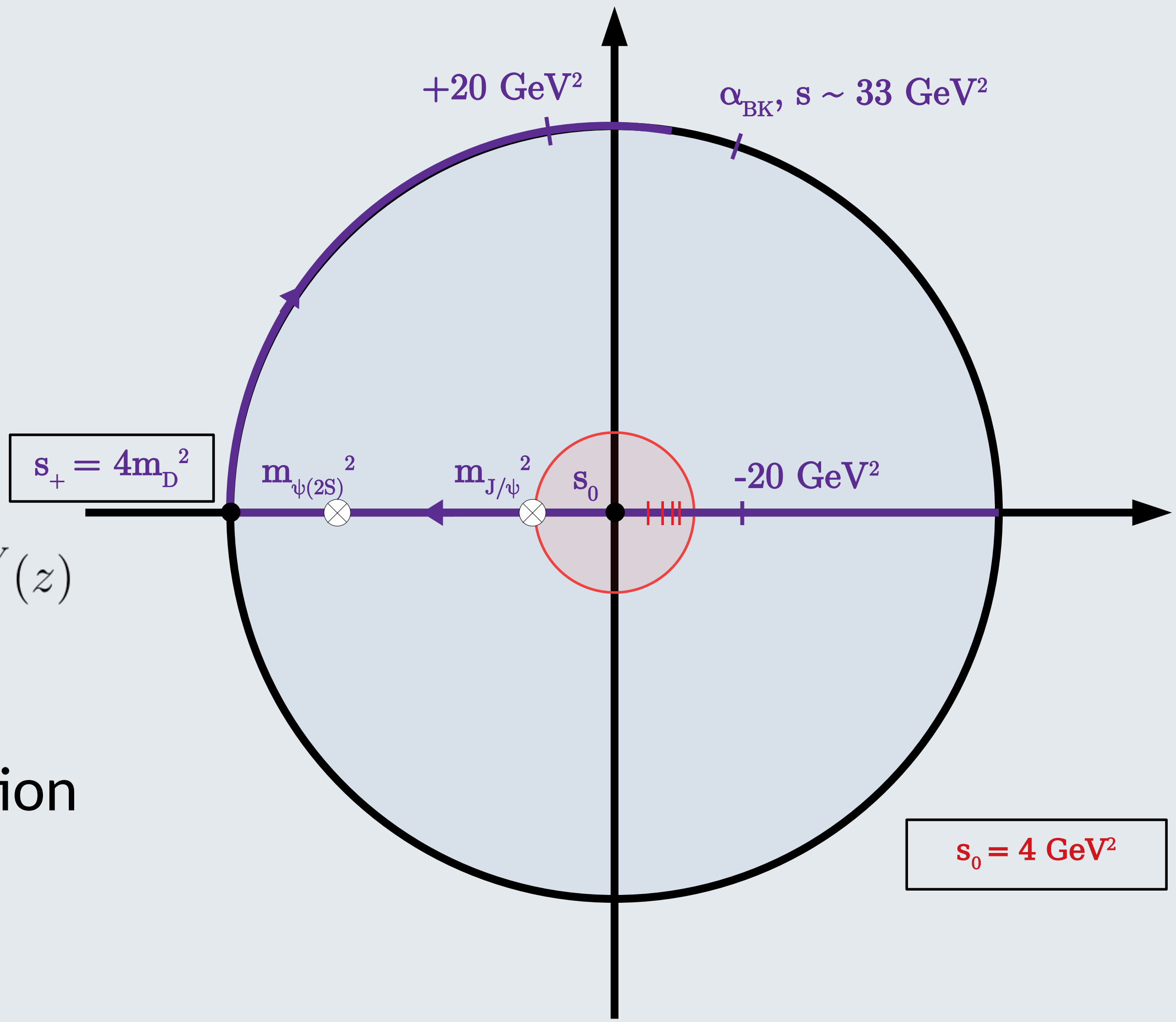
- **z-expansion**

$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M} z^n$$

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]



$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M} p_n^{B\to M}(z)$$

[Gubernari, van Dyk, Virto, 2020]

Dispersive bound

$$\hat{\mathcal{H}}_{\lambda}^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M}\, z^n$$

- In practice, $a_{\lambda,n}^{B\to M} = 0$ for $\mathrm{n} > \mathrm{N}$. **What is the truncation error?**

[Gubernari, van Dyk, Virto, 2020]

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- **Dispersive bound** (from the **Local** OPE)

$$1 > 2\int_{-\alpha_{BK}}^{+\alpha_{BK}} d\alpha \left|\hat{\mathcal{H}}_0^{B\to K}(e^{i\alpha})\right|^2 + \sum_\lambda \left[2\int_{-\alpha_{BK^*}}^{+\alpha_{BK^*}} d\alpha \left|\hat{\mathcal{H}}_\lambda^{B\to K^*}(e^{i\alpha})\right|^2 + \int_{-\alpha_{B_s\phi}}^{+\alpha_{B_s\phi}} d\alpha \left|\hat{\mathcal{H}}_\lambda^{B_s\to\phi}(e^{i\alpha})\right|^2\right]$$

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→ With **orthonormal polynomials:** $\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M}\, p_n^{B\to M}(z)$

$$\sum_{n=0}^{\infty}\left\{2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0}\left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2\right]\right\} < 1$$

Anticipating on the results:

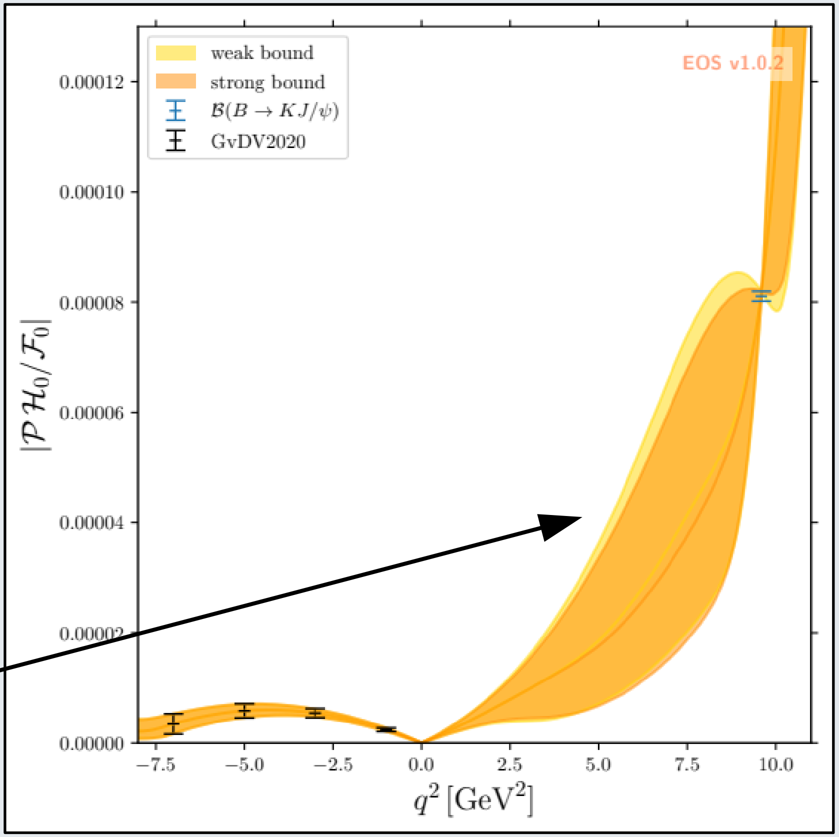
Expand up to **order 5:**

- 12 real parameters
- 8 constraints at negative q^2
- 1 constraint at $m_{J/\psi}^2$

→ 3 free parameters **constrained by the dispersive bound!**

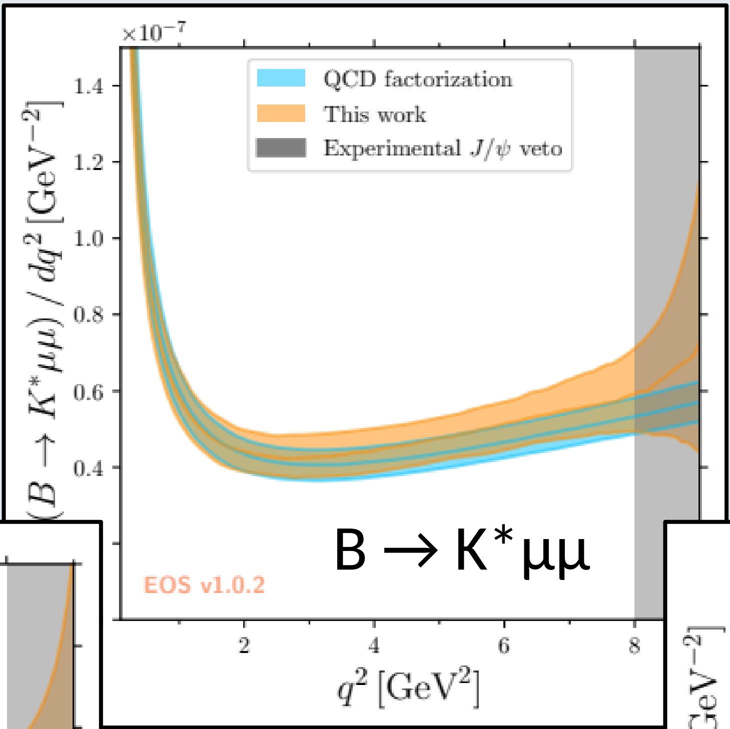
1) Controlled uncertainty in the physical region

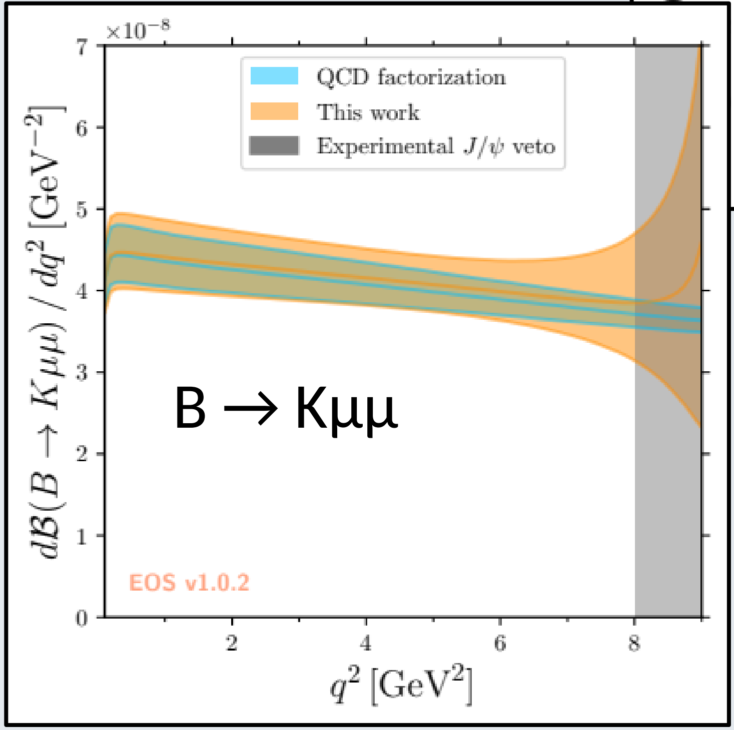
2) Adding an order in the expansion **doesn't increase this uncertainty!**

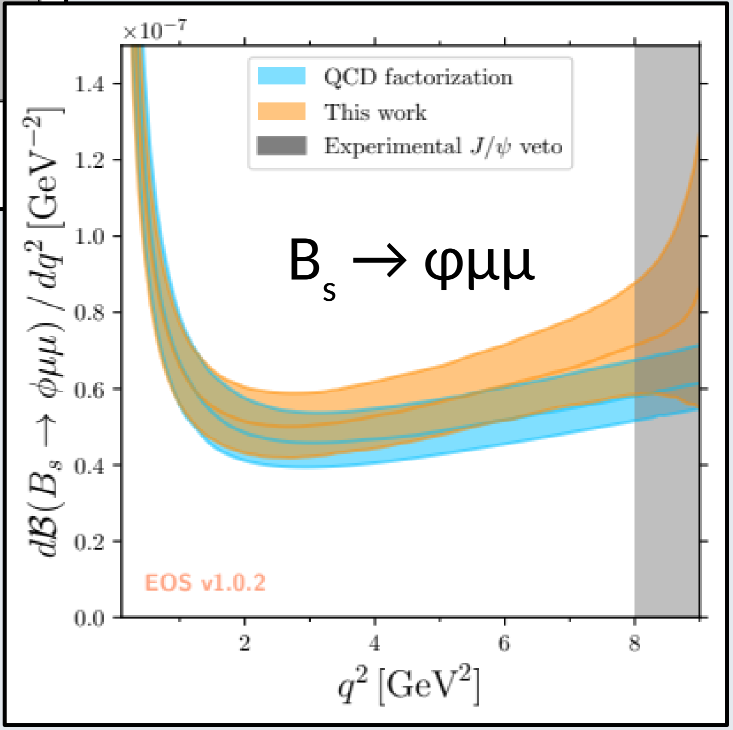


Comparison with QCDF

- **Good overall agreement** with QCDF
- Larger uncertainties especially near the J/ψ

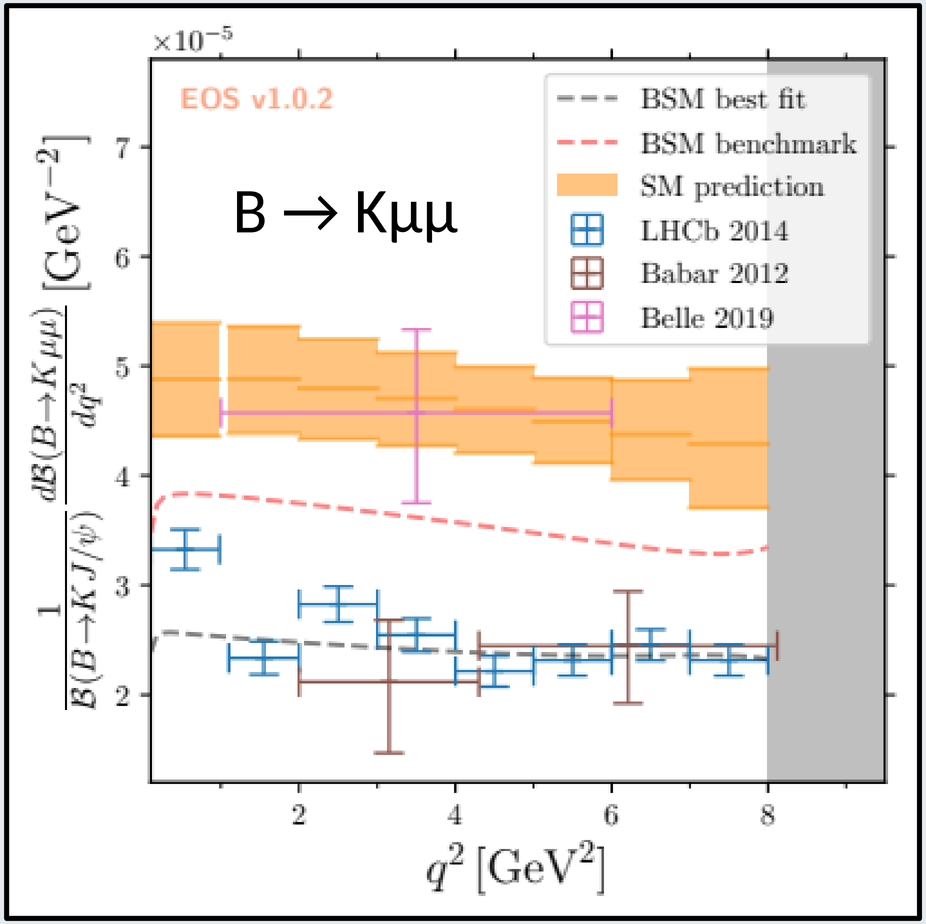


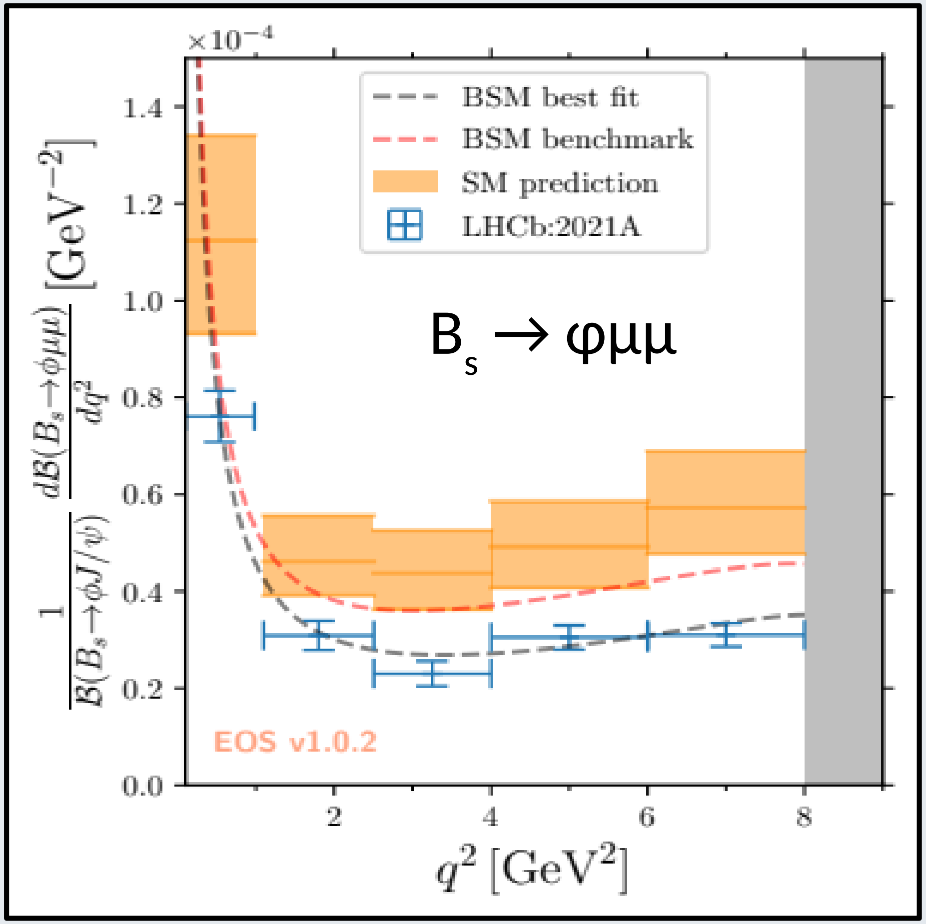




Updated (B)SM predictions

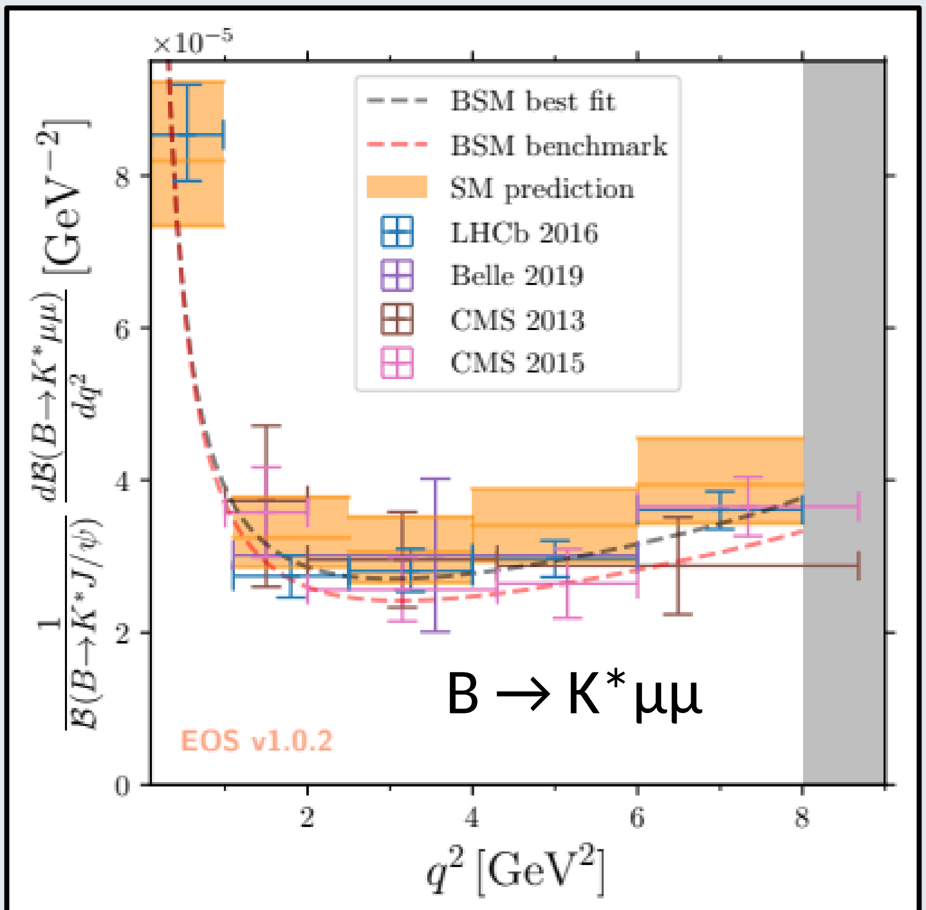
- We confirm the overall **tension** with experimental data

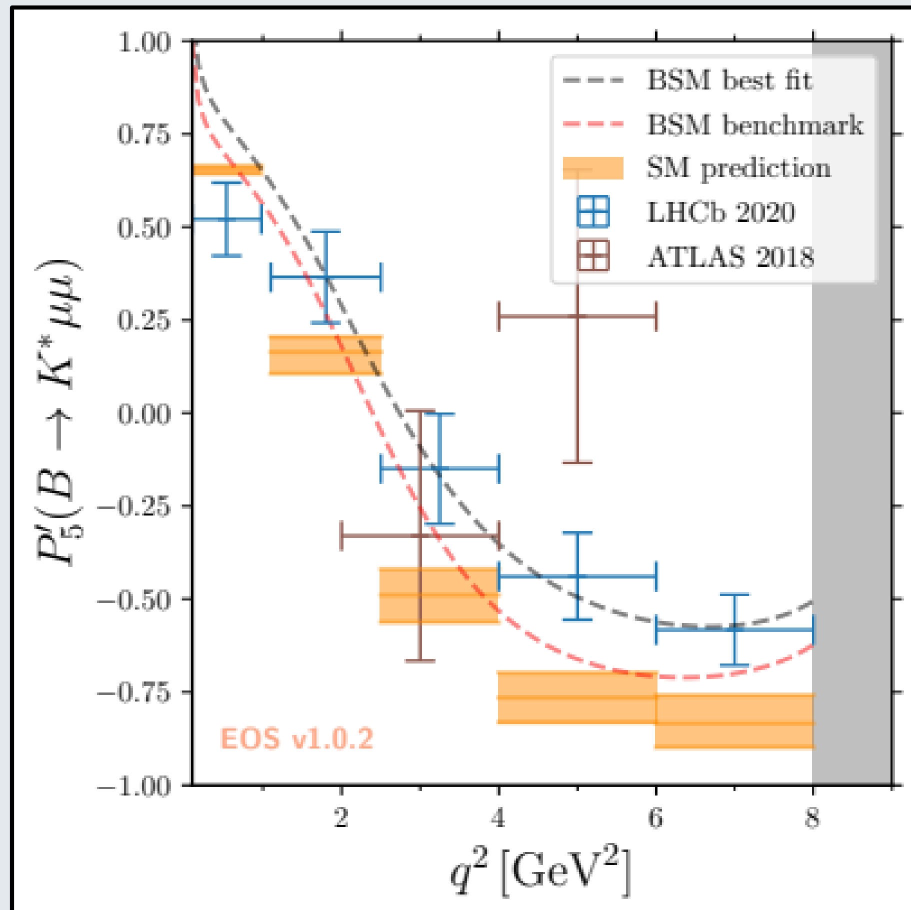




Updated (B)SM predictions

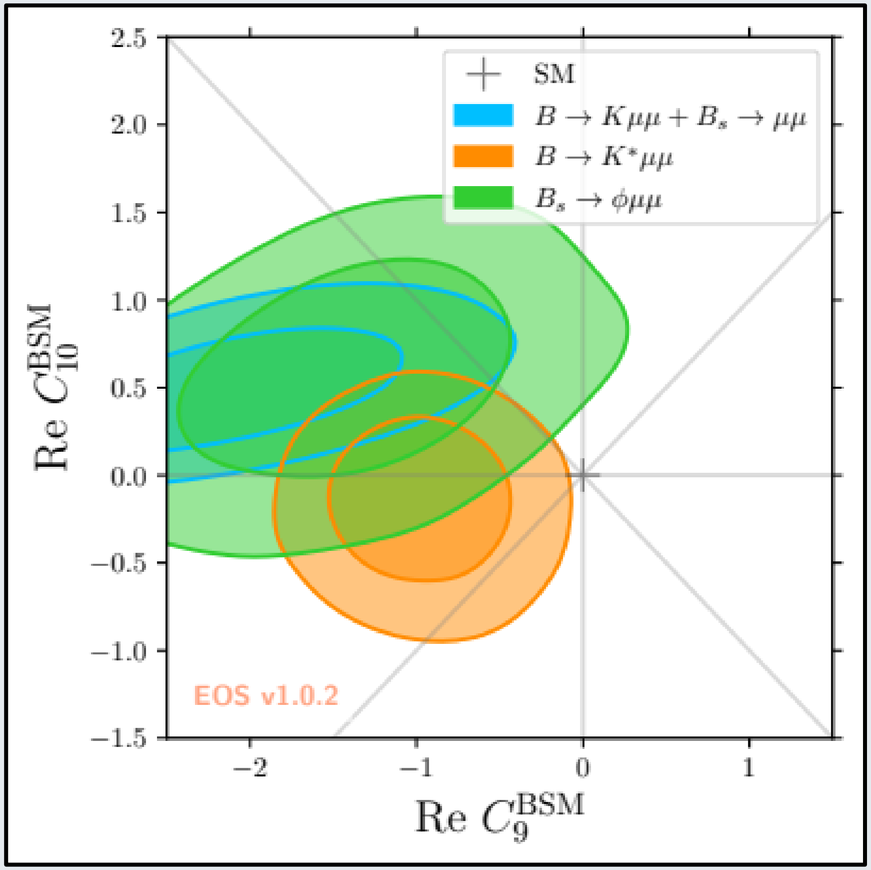
- For B → K*μμ the tension is smaller than in the literature due to **different approaches and inputs**





BSM analysis

- A combined BSM analysis would be **very CPU expensive** (130 nuisance parameters!)
- Fit **separately** C_9 and C_{10} for the three channels:
  $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$, $B \to K^*\mu^+\mu^-$ and $B_s \to \varphi\mu^+\mu^-$



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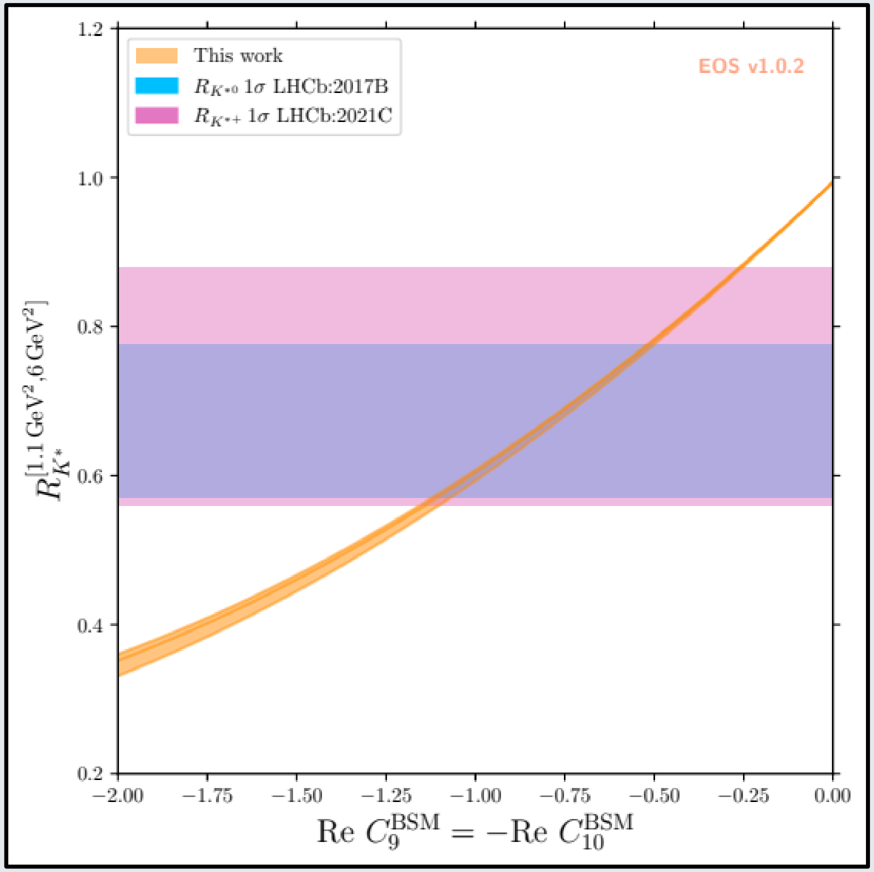


- Flavor universality-testing ratios

$$R_{K^{(*)}} = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)}$$

are **weakly sensitive to non-local contributions!**

Summary & Outlook

- We provide new **SM predictions of $b \to s\ell\ell$ observables** with
  - **Updated fit** to the *local* form-factors;
  - **Controlled uncertainties** on the *non-local* contributions.

- We performed a **(C_9, C_{10}) BSM analysis**, confirming the current trend.

- **What is next?**
  - Perform an **extended BSM analysis**;
  - Extend the prediction to **higher q^2** values (including the $\psi(2S)$)
  - Include **other channels** $\Lambda_b \to \Lambda \mu\mu$, …

Back-up

Putting everything together:

- The fit is performed in two steps...
  - Preliminary fits:
    - **Local** form factors:
      - BSZ parametrization (**8 + 19 + 19 parameters**)
      - LCSR + LQCD, **more in the backup**
    - **Non-local** form factors:
      - order 5 GvDV parametrization (**12 + 36 + 36 parameters**)
      - 4 points at negative q^2 + $B \to M\, J/\psi$ data

    $\to$ **130 nuisance parameters**
  - ‘Proof of concept’ fit to the WET’s **Wilson coefficients**
- ... using **EOS**:

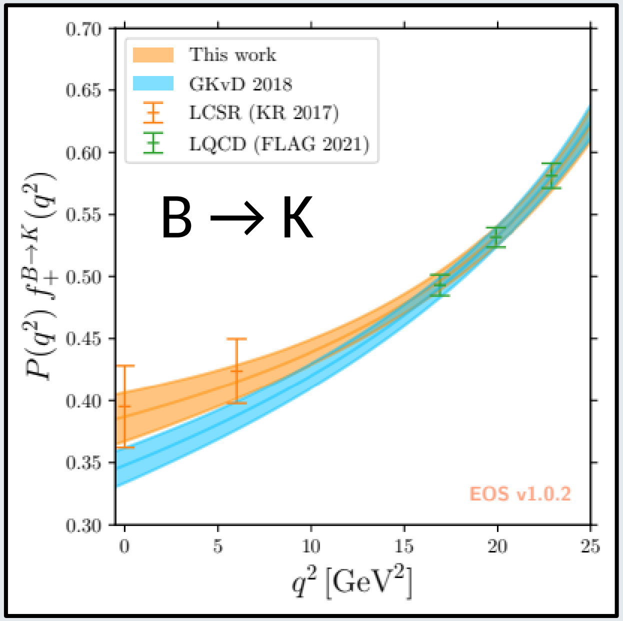
EOS is a software for a variety of applications in flavour physics. It is written in C++, but provides an interface to Python.

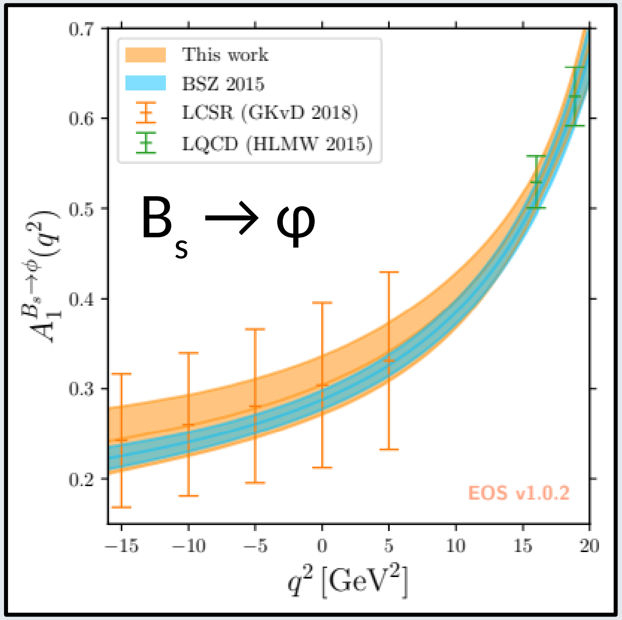
https://eos.github.io/

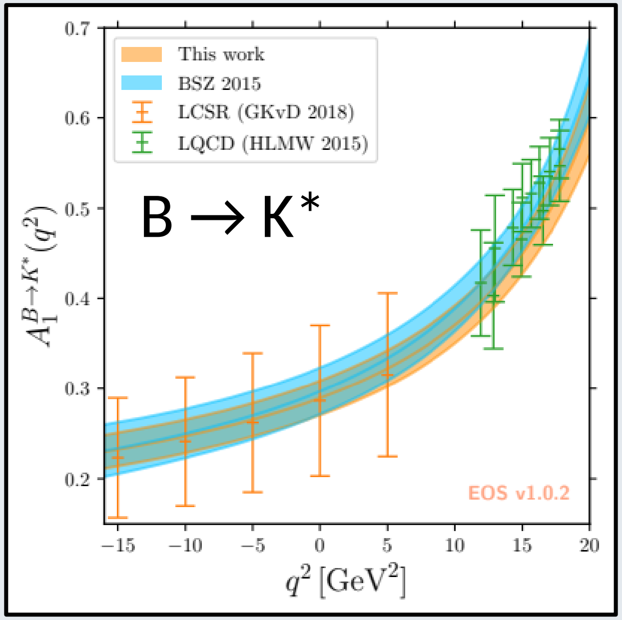
Fit to local form factors

Combined fit to LCSR and lattice:

- $B \to K$:
  - HPQCD'17; FNAL/MILC'17
  - Khodjamiriam and Rusov'17
- $B \to K^*$:
  - Horgan, Liu, Meinel and Wingate'15
  - Gubernari, Kokulu and van Dyk'18
- $B_s \to \varphi$:
  - Horgan, Liu, Meinel and Wingate'15
  - Gubernari, van Dyk and Virto'20







A few remarks

1. QCD Factorization [Beneke, Feldmann, Seidel, 2001 & 2004]
2. Theory uncertainties due to charm-loops **cancel in ratios observables** → "clean" observables
   Anomalies are not entirely due to charm-loops!
3. **Agreement** between "clean" and "not-so-clean" observables
   Charm-loops effects cannot be very large!
4. Naively **set theory uncertainty to 0 in H_λ**:

   → Significance of the C_9 vs. C_{10} fit rises from **~4σ** to **~8σ**!
   This talk is not a waste of time...
5. Theory **puzzles in $b \to s\bar{c}c$** [see e.g. Lyon, Zwicky, 2014]
   We need to be careful...

Data-driven (B)SM predictions